Energy and angle dependence of Primakoff cross section

Maximize $\quad d\sigma \propto \dfrac{\sin^2\theta}{Q^4} \quad$ WRT θ

$$Q^2 = -(q-k)^2 = +2qk - s$$

$$Q^2 = 2E_\gamma[E - k\cos\theta] - s \qquad \text{Assume } E \simeq E_\gamma$$

$$Q^2 = 2E_\gamma[E_\gamma - \sqrt{E_\gamma^2 - s}\cos\theta] - s$$

$$\frac{df}{d\theta} = \frac{2\sin\theta\cos\theta}{Q^4} - 2\frac{\sin^2\theta}{(Q^2)^3}\frac{dQ^2}{d\theta} = 0$$

$$\cos\theta - \frac{\sin\theta}{Q^2} \times 2E_\gamma\sqrt{E_\gamma^2 - s}\,\sin\theta = 0$$

work to order θ^2

$$\cos\theta - \frac{\sin^2\theta\, 2E_\gamma\sqrt{E_\gamma^2 - s}}{2E_\gamma^2\left[1 - \sqrt{1 - \frac{s}{E_\gamma^2}}\right] - s} = 0$$

$$\cos\theta - \frac{\sin^2\theta\, 2E_\gamma^2\left(1 - \frac{1}{2}\,{}^{s}/_{E_\gamma^2}\right)}{2E_\gamma^2\left[1 - \left(1 - \frac{1}{2}\,{}^{s}/_{E_\gamma^2} - \frac{1}{4}\,{}^{s^2}/_{E_\gamma^4}\right)\right] - s} = 0$$

$$\cos\theta - \frac{\sin^2\theta\, 2E_\gamma^2\left(1 - \frac{1}{2}\,{}^{s}/_{E_\gamma^2}\right)}{{}^{s^2}/_{2E_\gamma^2}} = 0$$

$$\cos\theta - \sin^2\theta \times \frac{2E_\gamma^2}{s}\left(\frac{1 - \frac{1}{2}\,{}^{s}/_{E_\gamma^2}}{{}^{s}/_{2E_\gamma^2}}\right) = 0$$

$$\cos\Theta - \sin^2\Theta \times 2\frac{E_\gamma^2}{s}\left[\frac{2E_\gamma^2}{s} - 1\right] = 0$$

$$1 - \frac{\Theta^2}{2} - \Theta^2 \frac{4E_\gamma^4}{s^2} = 0$$

$$1 - 4\frac{E_\gamma^4}{s}\Theta^2 = 0$$

$$\boxed{\Theta_{max} = \frac{s}{2E_\gamma^2}}$$

Find energy dependence of peak differential cross section

$$d\sigma \propto \frac{E_\gamma^4}{s^{3/2}Q^4}\sin^2\Theta$$

$$Q^2 = 2E_\gamma\left[E - \sqrt{E^2 - s}\cos\Theta\right] - s$$

$$Q^2 = 2E_\gamma E\left[1 - \sqrt{1 - s/E^2_\gamma}\cos\Theta\right] - s$$

$$Q^2 = 2E_\gamma E\left[1 - \left(1 - \tfrac{1}{2}s/E^2 - \tfrac{1}{4}s^2/E^4\right)\left(1 - \tfrac{1}{2}\left(\tfrac{s}{2E_\gamma^2}\right)^2\right)\right] - s$$

$$Q^2 = 2E_\gamma E\left[\frac{s}{2E^2} + \left(\frac{s}{2E^2}\right)^2 + \frac{1}{2}\left(\frac{s}{2E_\gamma^2}\right)^2\right] - s$$

$$Q^2 = s\left(\frac{E_\gamma}{E} - 1\right) + \frac{1}{2}E_\gamma E s^2\left(\frac{1}{E^4} + \frac{1}{2}\frac{1}{E_\gamma^4}\right)$$ (see next page)

$$d\sigma_{peak} \propto \frac{1}{s^{3/2}}\frac{E_\gamma^4}{\left(\frac{s^2}{E_\gamma^2}\right)^2}\left(\frac{s}{2E_\gamma^2}\right)^2$$

$$\boxed{d\sigma_{peak} \propto \frac{E_\gamma^4}{s^{7/2}} = \frac{E_\gamma^4}{m^7}}$$

Integrated cross section

$$\sigma \sim \frac{E_\gamma^4}{m^7} \times 2\pi\left(1 - \cos\Theta_{max}\right)$$

$$\sigma \propto \frac{E_\gamma^4}{m^7} \times \left[1 - \left(1 - \frac{1}{2} \left(\frac{m^2}{2 E_\gamma^2} \right)^2 \right) \right]$$

$$\boxed{\sigma \propto \frac{1}{m^3}}$$

$$t = (q - k)^2 = (p - m)^2$$

$$t = m_T^2 - 2 E_T M_T + m_T^2$$

$$t = 2 m_T^2 - 2 (M_T + kE) m_T$$

$$t = -2 m_T kE$$

$$Q^2 = 2 m_T (E_\gamma - E)$$

$$\frac{Q^2}{2 m_T E_\gamma} = (1 - E/E_\gamma)$$

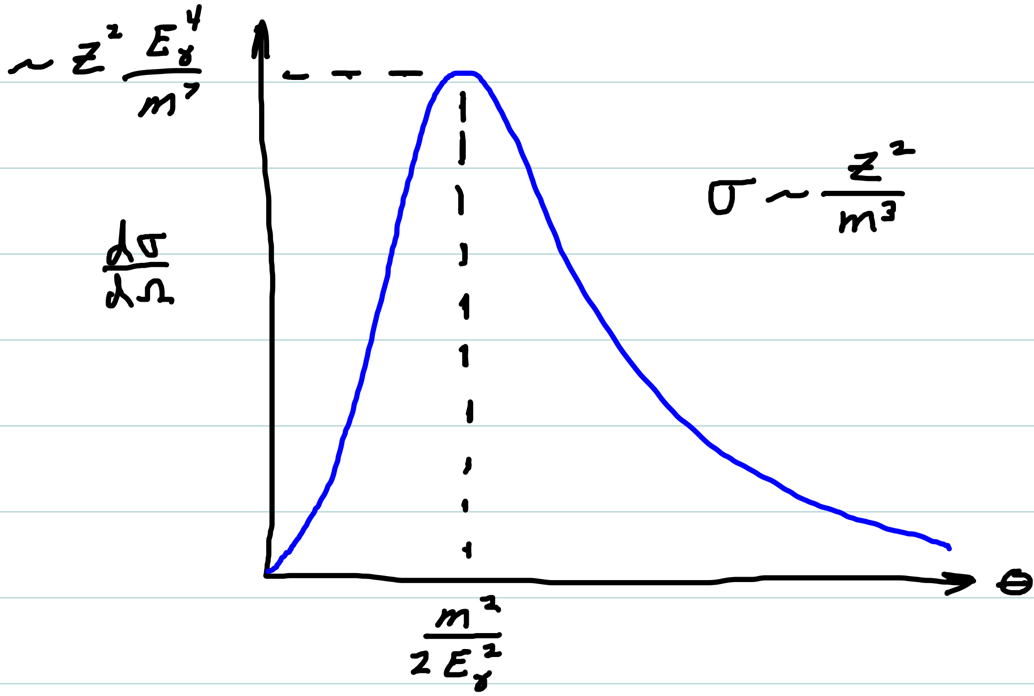
$$\frac{E_\gamma}{E} = \frac{1}{1 - \frac{Q^2}{2 m_T E_\gamma}} = 1 + \frac{Q^2}{2 m_T E_\gamma}$$

$$Q^2 = s \frac{Q^2}{2 m_T E_\gamma} + \frac{3}{4} \left(\frac{s}{E_\gamma} \right)^2 = \frac{3}{4} \frac{s^2}{E_\gamma^2}$$

Primakoff Differential Cross Section

$$\frac{d\sigma}{d\Omega} = \Gamma(\pi^0 \to \gamma\gamma)\, \frac{8\alpha Z^2}{m^3}\, \beta^3 \frac{E_\gamma^4}{Q^4}\, |F(Q^2)|^2 \sin^2\theta$$

Assume $F(Q) \simeq 1$



$$Q^2_{max} \simeq \frac{3}{4}\frac{m^4}{E_\gamma^2}$$