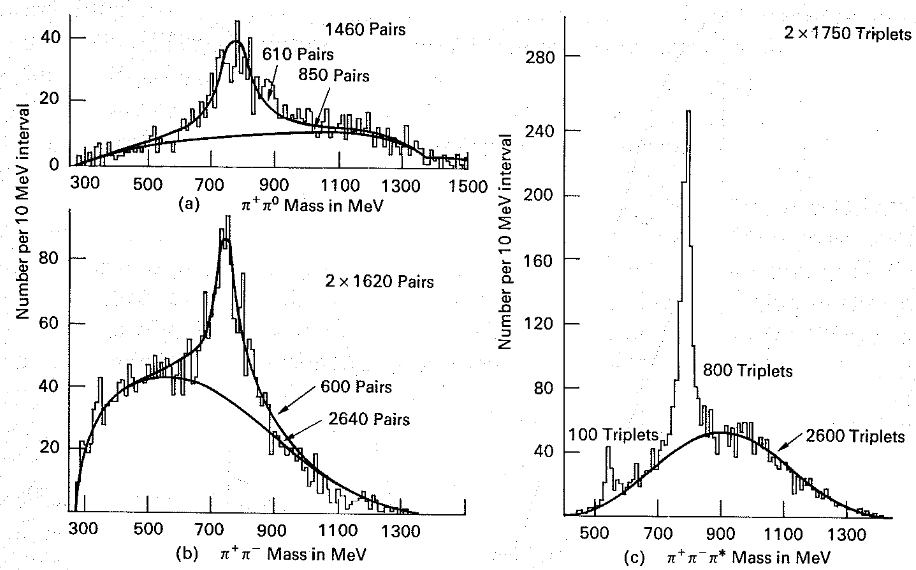


**Fig. 7.17** Mass distributions of (a) $\pi^+\pi^0$ pairs from reaction (7.34a), (b) $\pi^+\pi^-$ pairs from reaction (7.34b). The smooth curves indicate the distributions expected from phase space. (c) The $\pi^+\pi^-\pi^0$ invariant mass spectrum from reaction (7.34c). The narrow peak at 785 MeV corresponds to the $\omega \to 3\pi$ resonance, and that at 550 MeV to $\eta \to 3\pi$. (From Alff *et al.*, 1962.)

and by several other groups. Figure 7.17 shows results from a study in a hydrogen bubble chamber of the following reactions, using incident pions of momentum 1.6 to 1.9 GeV/$c$:

$$\pi^+ + p \to \pi^+ + \pi^0 + p \tag{7.34a}$$

$$\to \pi^+ + \pi^+ + \pi^- + p \tag{7.34b}$$

$$\to \pi^+ + \pi^+ + \pi^- + \pi^0 + p. \tag{7.34c}$$

The mass distributions of $\pi^+\pi^0$ pairs from reaction (7.34a) and $\pi^+\pi^-$ pairs from (7.34b) show a broad peak centered at 765 MeV, with a width about 120 MeV, attributed to the decay $\rho \to 2\pi$. Experiments have failed to detect a similar peak in the $\pi^+\pi^+$ mode.

These results indicate an isospin assignment $I = 1$ for the $\rho$-meson. Since we have two identical bosons in the final state, $I = 1$ implies that the spatial wave function of the pion pair must be antisymmetric. The simplest possibility is $l = 1$, corresponding to the spin-parity quantum numbers for the $\rho$-meson: $J^P = 1^-$.

A two-body decay is less informative than a three-body decay process, such as was analyzed in determining the quantum numbers of the $\omega$-meson. To prove the above spin-parity assignment, we have to rely on observations relating to the production process itself. The easiest way to determine the spin is to consider a special class of "peripheral" events, with small momentum transfer to the nucleon. For this purpose, consider the reactions

$$\pi^- + p \to \pi^- + \pi^0 + p \tag{7.35a}$$

$$\to \pi^+ + \pi^- + n. \tag{7.35b}$$

As discussed in Section 7.8, these events can be described by single pion exchange, as indicated diagrammatically in Fig. 7.18(a). If we now look in the rest frame of the $\rho$-meson [Fig. 7.18(b)], we are essentially observing the collision of the incident $\pi^-$ with a virtual $\pi^+$ of the meson cloud surrounding the nucleon:

$$\pi^- + \pi^+_{(\text{virtual})} \to \rho \to \pi^+ + \pi^-.$$

If we choose the axis of quantization, $z$, along the incident pion direction, it is clear that in the initial, and hence also the final, state the $z$-component of angular momentum $m = 0$, since pions have zero spin. Thus, the angular distribution of the scattered pions will be

$$F(\theta) = (Y_l^m)^2 = [P_J^0(\cos\theta)]^2,$$

where $J$ is the $\rho$-spin and, for $m = 0$, the azimuthal distribution $e^{im\phi}$ is isotropic.

The measured angular distributions for $\pi^-\pi^0$ pairs in (7.35a) is found to have the form

$$F(\theta) = A + B\cos\theta + C\cos^2\theta. \tag{7.36}$$

The fact that no terms higher than $\cos^2\theta$ appear is itself indicative that $J_\rho = 1$. The form (7.36) is interpreted as a coherent superposition of two amplitudes; $A_1$ due to a $J = 1$ resonance, and $A_0$ due to an $S$-wave $\pi\pi$ interaction (of $I = 2$, since $J = 0$) forming a nonresonant background term. Then

$$\begin{aligned} F(\theta) &= |A_0 + A_1 \cos\theta|^2 \\ &= |A_0|^2 + |A_1|^2\cos^2\theta + 2\,\text{Re}\,A_0A_1^*\cos\theta, \end{aligned} \tag{7.37}$$

where we have used the fact that $P_0^0 = 1$, $P_1^0 = \cos\theta$. Now, as indicated in Fig. 7.11, the amplitude $A_1$ must become purely imaginary at the $\rho$-peak, so

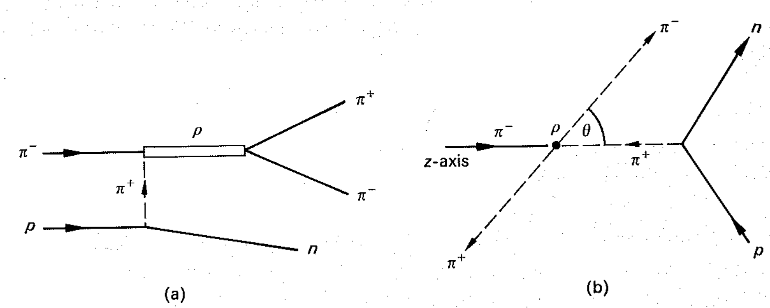


**Figure 7.18**

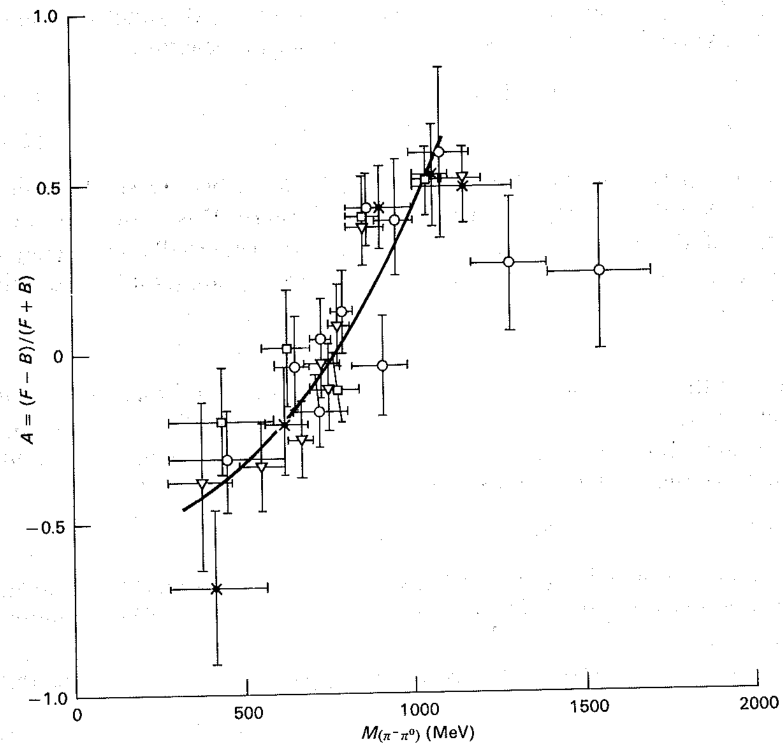


**Fig. 7.19** Forward-backward ratio in the pion angular distribution as a function of $M_{\pi^0\pi^-}$ in reaction (7.35a), as measured by various experiments for incident pion momenta from 2.75 to 6 GeV/$c$. The forward-backward asymmetry goes to zero at the $\rho$-mass ($M_{\pi^-\pi^0}$ = 765 MeV). (After Baton *et al.*, 1965.)

the interference ($\cos\theta$) term will change sign as one goes through it. This can be seen in terms of the forward-backward ratio of Fig. 7.19.

The assumption of single-pion exchange, necessary to obtain the above result, is borne out by comparing the rates for reactions (7.35a) and (7.35b). Using the table of Clebsch-Gordan coefficients (Appendix C) for adding two isospins of unity, one finds

$$\phi(\pi^-\pi^0) = \frac{1}{\sqrt{2}}\psi(1, -1) + \frac{1}{\sqrt{2}}\psi(2, -1),$$

$$\phi(\pi^+\pi^-) = \frac{1}{\sqrt{3}}\psi(0, 0) + \frac{1}{\sqrt{2}}\psi(1, 0) + \frac{1}{\sqrt{6}}\psi(2, 0),$$

where $\psi(I, I_3)$ denotes a particular isospin state and its $z$-component. For

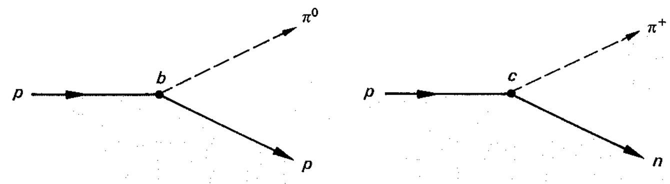


**Figure 7.20**

$I = 1$, we have

$$\frac{\sigma(\pi^- + \pi^0 \to \rho^- \to \pi^- + \pi^0)}{\sigma(\pi^+ + \pi^- \to \rho^0 \to \pi^+ + \pi^-)} = \left| \frac{\frac{1}{\sqrt{2}}\psi(1,-1)}{\frac{1}{\sqrt{2}}\psi(1,0)} \right|^2 = 1.$$

We must remember that one pion in the initial state is virtual. Referring to Section 3.11, we found for the ratio of the couplings $b/c = 1/\sqrt{2}$. Reactions (7.35a) and (7.35b) involve the left- and right-hand diagrams of Fig. 7.20 respectively, so that our final predicted ratio is

$$\frac{\sigma(\pi^- p \to \rho^0 n)}{\sigma(\pi^- p \to \rho^- p)} = \left(\frac{c}{b}\right)^2 = 2.$$

The observed ratio is $1.8 \pm 0.2$, in agreement with this.

It may also be remarked that the angular distribution of pairs from (7.35b) does *not* follow the form of Fig. 7.19; i.e. the $\cos\theta$ term is not zero at the $\rho$-peak (see Fig. 7.21). This implies that the $J = 0$ amplitude, $A_0$, *also* has an imaginary component, corresponding to resonant behavior. In this case, since $I_3 = 0$, this must have the quantum numbers $J = 0, I = 0$ (not $I = 2$, as for the $\pi^-\pi^0$ case, which is a nonresonant amplitude). This possible resonance is referred to as the $\varepsilon$-meson. Its existence has not yet clearly been established. It could be more clearly seen in the $\pi^0\pi^0$ state, but this is experimentally difficult.

To summarize, the $\rho$-meson has $I = 1, G = +1, J^P = 1^-$. Its production in pion-nucleon interactions may be pictured in terms of collision of the incident pion with a single virtual pion from the nucleon target. This is discussed further in Section 7.8.

It has been mentioned in Chapter 5 that the $\rho$-meson may also be produced in electron-positron colliding beam experiments. In such experiments, the width of the $\rho$-resonance appears to be somewhat smaller ($\Gamma \approx 100$ MeV) than that found for $\rho$'s produced in strong interactions such as (7.34), where $\Gamma \approx 120$ to 140 MeV. The difference, if real, has to be attributed to the effect of final state interactions in strong production, where other hadrons are involved. Put in another way, the Breit-Wigner amplitude is the Fourier transform of an exponential time pulse (integrated from $t = 0$ to $t = \infty$),