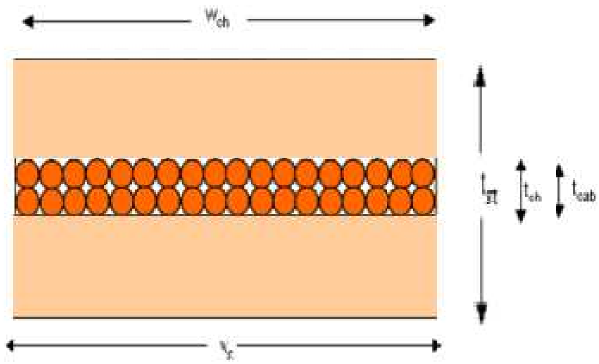
Single coil Quench Analysis- Hall D (Solenoid)

Probir Kumar Ghoshal and Renuka Rajput-Ghoshal (16th May 2013)
March 2013_**Copper Channel as built**
Version 6.0

This worksheet assumes that all the stored energy of the magnet is dumped into single coil.
For multi-coil system calculations should be repeated for all coils.
This document is based on all the formulas given in Martin Wilson's book (used for Magnet design)
This worksheet calculates the following parameters for quenching coils:

1. *U-Function and how quickly the coils must discharge to avoid thermal damage*
2. *MPZ and MPZ energy, measures of conductor stability*
3. *Quench Velocities and quench propagation times*
4. *Quench decay time and predicted peak temperatures & voltages*
5. *It is important that these preliminary calculations are supplimented with a proper quench model if the magnet is complex or has a large stored energy.*
6. *The is PRIMARILY CARRIED OUT BASE ON **COIL #1A of HALL D solenoid***



Stabilizer width $w_{st} := 7.62mm$

Stabilizer thickness $t_{st} := 2.197mm$

Channel width $w_{ch} := w_{st} = 7.62\,mm$

Chanel thickness $t_{ch} := 0.9398mm$

$RRR_m := 100$ $RRR_{st} := 120$ RRR value for the matrix=100
RRR value for Stabilizer=120

Total conductor area including copper channel Acon

$A_{con} := 2w_{st} \cdot t_{st} + w_{ch} \cdot t_{ch}$ $A_{con} = 40.644\,mm^2$

Wire diameter $d_w := 0.127mm$ This is NbTi wire diameter

Number of wires $N_w := 87$ Number of NbTi wire in conductor

Material in the conductor $mat := 4$ This is matrix to superconductor ratio

Wire Area $A_w := N_w \cdot \left(\frac{\pi}{4}\right) \cdot d_w^{\,2} = 1.102\,mm^2$ No of starnds

Channel occupied by cable $A_{ch} := w_{ch} \cdot t_{ch} = 7.161\,mm^2$ $N_s := \mathrm{round}\left(\frac{A_{ch}}{A_w}\right) = 6$

Wire copper area $A_{wcu} := N_s \cdot A_w \cdot \frac{mat}{1 + mat} = 5.29\,mm^2$

Wire NbTi Area $A_{nt} := N_s \cdot A_w \cdot \frac{1}{1 + mat} = 1.323\,mm^2$

Solder area $A_{vo} := A_{ch} - 6 \cdot A_w = 0.549\,mm^2$

Insulation radial thickness $t_i := 1.02mm$

Inter pancake insulation $t_{ip} := 2.34mm$

Ground plane insulation $t_{ig} := 5mm$

Width unit cell $w_u := w_{st} + 2 \cdot t_i = 9.66\,mm$

Unit cell thickness $t_u := 2t_{st} + t_{ch} + 2 \cdot t_i = 7.374\,mm$

Unit cell area $A_u := w_u \cdot t_u = 71.231\,mm^2$

Insulation area $A_i := A_u - A_{con} = 30.587\,mm^2$

Stabalizer area $A_{st} := A_{con} - A_{ch} = 33.482\,mm^2$

Insulation area is (unit cell area -conductor area) and stabilizer area is (conductor area-channel area)

Total Copper area $A_{Cut} := A_{st} + 6 \cdot A_{wcu} = 65.222\,mm^2$

Over Unit cell $\lambda_{st} := \frac{A_{st}}{A_u} = 0.47$ $\lambda_{wcu} := \frac{A_{wcu}}{A_u} = 0.074$

$\lambda_{cu} := \lambda_{st} + \lambda_{wcu} = 0.544$ $\lambda_{vo} := \frac{A_{vo}}{A_u} = 7.704 \times 10^{-3}$

$\lambda_{nt} := \frac{A_{nt}}{A_u} = 0.019$ $\lambda_i := \frac{A_i}{A_u} = 0.429$

Cu to Sc ratio $CuSc\_R := \frac{\lambda_{cu}}{\lambda_{nt}} = 29.317$

check if all the ratio are correct $\lambda_{cu} + \lambda_{nt} + \lambda_{vo} + \lambda_i = 1$

Winding Composition

The following materials make up the conductor:

Area of NbTi: $A_{NbTi} := A_{nt}$ $A_{NbTi} = 1.323 \cdot mm^2$

Area of Copper: $A_{Cu} := A_{wcu}$ $A_{Cu} = 5.29 \cdot mm^2$

Area of Copper channel: $A_{CuCh} := A_{st} = 33.482\,mm^2$

Area of solder $A_{vo} = 0.549\,mm^2$

Area of insulation $A_i = 30.587\,mm^2$

Coil Parameters

Coil dimensions: Coil Parameters

The coil block dimensions for the smallest coil in present solenoid design are:

$R_1 := 1018 \cdot mm$ $\quad \Delta R := 174 \cdot mm$

$R_2 := R_1 + \Delta R$ $\quad R_2 = 1.192 \times 10^3 \, mm$

$Z_1 := 0mm$ $\quad \Delta Z := 117.14mm$

$Z_2 := Z_1 + \Delta Z$ $\quad Z_2 = 117.14 \, mm$

$L_{pinner} := 2\pi \cdot R_1$ $\quad L_{pouter} := 2\pi \cdot R_2$

$L_{pinner} = 6.396 \times 10^3 \, mm$ $\quad L_{pouter} = 7.49 \times 10^3 \, mm$

$L_{paverage} := \dfrac{(L_{pinner} + L_{pouter})}{2}$ $\quad L_{paverage} = 6.943 \times 10^3 \, mm$

Coil dimensions: Layer to layer: $\quad H_L := R_2 - R_1$ $\quad H_L = 174 \cdot mm$

Turn to turn: $\quad W_T := Z_2 - Z_1$ $\quad W_T = 117.14 \cdot mm$

Numbers of turns and layers:

$N_T := \text{round}\left(\dfrac{W_T}{w_{st} + t_{ip}}\right)$ $\quad N_T = 12$

$N_L := \text{round}\left(\dfrac{H_L}{2t_{st} + t_{ch} + 2t_i}\right)$ $\quad N_L = 24$

$N_C := N_T \cdot N_L = 288$ $\quad AT := 432000 \cdot A$

$L_p := L_{paverage} = 6.943 \times 10^3 \, mm$

The coil unit cell area is then $\quad A_c := A_u$ $\quad A_c = 71.231 \cdot mm^2$

and the area of epoxy glass is $\quad A_{EG} := A_c - A_{NbTi} - A_{Cu} - A_{CuCh} - A_{vo}$

$A_{EG} = 30.587 \cdot mm^2$

$\lambda_{NbTi} := \dfrac{A_{NbTi}}{A_c}$ $\quad \lambda_{Cu} := \dfrac{A_{wcu}}{A_c}$ $\quad \lambda_{CuCh} := \dfrac{A_{st}}{A_c}$ $\quad \lambda_{ins} := \dfrac{A_{EG}}{A_c}$

$\lambda_{NbTi} = 0.019$ $\quad \lambda_{Cu} = 0.074$ $\quad \lambda_{CuCh} = 0.47$ $\quad \lambda_{ins} = 0.429$ $\quad \lambda_{vo} = 7.704 \times 10^{-3}$

Operating Conditions:

All the highlighted values in this section needs to be changed for the operting condition of the coil. Maximum coil field is the maximum field in the coil being considered here. The effective total inductance is the inductance of all the coils connected in the circuit.

Operating current: $I_0 := \frac{AT}{N_C} = 1.5 \times 10^3 \cdot A$

$J_0 := \frac{I_0}{A_c}$ $J_0 = 21.058 \cdot A \cdot mm^{-2}$

Maximum coil field: $B_{max} := 2.97 \cdot T$ Maximum field value

Operating temperature: $\theta_0 := 4.603 \cdot K$ Operating temperature of the magnet

Effective total inductance acting on the quenching coil:

$(L_{tot} := 28.71 \cdot H)$

Stored energy: $E_{st} := \frac{1}{2} \cdot L_{tot} \cdot I_0^2$ $E_{st} = 3.23 \times 10^7 \cdot J$

U-Function

Function U contains only the properties of material used in the winding. This function is used to calculate the maximum temperature in terms of initial current density and a characteristic time T_d for the current decya following a quench.

Heat balance assuming adiabatic conditions:

$$J(t)^2 \cdot \rho(\theta) \cdot dt = \gamma \cdot C(\theta)\, d\theta$$

Chapter 9 of MNW's book equation 9.3

If the current density remains constant at the initial value for the whole of the decay time:

$$J_0^2 \cdot T_d = \int_{\theta_0}^{\theta_{max}} \frac{\gamma \cdot C(\theta)}{\rho(\theta)}\, d\theta = U(\theta)$$

Chapter 9 of MNW's book equation 9.4

Specific Heats

Reference:J:\CLAS12 Magnet\Expe_Pers\Quench_RRG_PKG\Material data\NbTi specific heat.xmcd

Reference:J:\CLAS12 Magnet\Expe_Pers\Quench_RRG_PKG\Material data\Copper specific heat.xmcd

Reference:J:\CLAS12 Magnet\Expe_Pers\Quench_RRG_PKG\Material data\Epoxy Glass specific heat.xmcd

Densities

$\gamma_{NbTi} := 6140 \cdot kg \cdot m^{-3}$ $\gamma_{EG} := 1740 \cdot kg \cdot m^{-3}$

$\gamma_{Cu} := 8930 \cdot kg \cdot m^{-3}$ $\gamma_{vo} := 1 \cdot kg \cdot m^{-3}$

Overall Winding Density

$$\gamma_{av} := \frac{A_{CuCh} \cdot \gamma_{Cu} + A_{Cu} \cdot \gamma_{Cu} + A_{NbTi} \cdot \gamma_{NbTi} \cdots + A_{EG} \cdot \gamma_{EG} + A_{vo} \cdot \gamma_{vo}}{A_c}$$

$$\gamma_{av} = 5.722 \times 10^{3} \cdot kg \cdot m^{-3}$$

Average winding density is very close to copper density because of the higher copper fraction.

Average Specific Heat Capacity

$$C_{av}(\theta) := \frac{A_{CuCh} \cdot \gamma_{Cu} \cdot C_{Cu}(\theta) + A_{Cu} \cdot \gamma_{Cu} \cdot C_{Cu}(\theta) + A_{NbTi} \cdot \gamma_{NbTi} \cdot C_{NbTi}(\theta) \cdots + A_{EG} \cdot \gamma_{EG} \cdot C_{eg}(\theta)}{A_{CuCh} \cdot \gamma_{Cu} + A_{Cu} \cdot \gamma_{Cu} + A_{NbTi} \cdot \gamma_{NbTi} \cdots + A_{EG} \cdot \gamma_{EG} + A_{vo} \cdot \gamma_{vo}}$$

$C_{Cu}(4.5 \cdot K) = 0.164 \cdot J \cdot kg^{-1} \cdot K^{-1}$ $\quad C_{NbTi}(4.5 \cdot K) = 0.108 \frac{1}{kg \cdot K} \cdot J$

$C_{eg}(4.5 \cdot K) = 0.047 \frac{1}{kg \cdot K} \cdot J$

$C_{av}(4.5 \cdot K) = 0.147 \cdot J \cdot kg^{-1} \cdot K^{-1}$ $\quad C_{av}(295 \cdot K) = 595.105 \cdot J \cdot kg^{-1} \cdot K^{-1}$

$C_{Cu}(\theta_0) = 0.173 \frac{1}{kg \cdot K} \cdot J$ $\quad C_{NbTi}(\theta_0) = 0.115 \frac{1}{kg \cdot K} \cdot J$

$C_{eg}(\theta_0) = 0.05 \frac{1}{kg \cdot K} \cdot J$

$C_{av}(\theta_0) = 0.156 \cdot J \cdot kg^{-1} \cdot K^{-1}$ $\quad C_{av}(295 \cdot K) = 595.105 \cdot J \cdot kg^{-1} \cdot K^{-1}$

Resistivities

Reference:J:\CLAS12 Magnet\Expe_Pers\Quench_RRG_PKG\Material data\Copper resistivity.xmcd

assume the resistivity of NbTi and solder to be 100 times that of copper:

$\rho_{NbTi}(\theta) := \rho_{Cu}(\theta, 100) \cdot 100$ $\quad \rho_{Cu}(20 \cdot K, 50) = 3.518 \times 10^{-10} \cdot \Omega \cdot m$

$\rho_{so}(\theta) := \rho_{Cu}(\theta, 100) \cdot 100$

Cu magnetoresistance factor $\quad m_B := 4 \cdot 10^{-11} \cdot \frac{\Omega \cdot m}{T}$ $\quad$ Martin Wilson data

$\rho_{CuB}(\theta, RRR) := \rho_{Cu}(\theta, RRR) + m_B \cdot B_{max}$

$\rho_{CuB}(\theta_0, 100) = 2.888 \times 10^{-10} \cdot \Omega \cdot m$

Average Resistivity

$$\rho_{av}(\theta) := \frac{A_{CuCh} + A_{Cu} + A_{NbTi}}{\frac{A_{CuCh}}{\rho_{CuB}(\theta,200) + \Delta\rho_{Cu}(B_{max})} + \frac{A_{Cu}}{\rho_{CuB}(\theta,100) + \Delta\rho_{Cu}(B_{max})} + \frac{A_{NbTi}}{\rho_{NbTi}(\theta)}}$$

$$\frac{\rho_{av}(100 \cdot K)}{A_{con} + A_{NbTi} + A_{vo}} = 1.023 \times 10^{-4} \cdot \Omega \cdot m^{-1}$$

Effective conductor resistance per unit length.

$$\frac{\rho_{av}(100 \cdot K)}{A_{con} + A_{NbTi} + A_{vo}} \cdot L_p = 7.101 \times 10^{-4} \Omega$$

resistance per turn.

$$\frac{\rho_{av}(100 \cdot K)}{A_{con} + A_{NbTi} + A_{vo}} \cdot L_p \cdot N_T \cdot N_L = 0.204 \Omega$$

resistance of the coil.

Average resistivity of Cu calculation from MW's based on Room temperature value only!:

$$\rho_{RT} := 1.678 \cdot 10^{-8} \Omega \cdot m$$

$$\rho_{ow} := \frac{\rho_{RT}}{RRR_m} = 1.678 \times 10^{-10} \cdot \Omega \cdot m$$

$$\rho_{cBw} := \rho_{ow} + m_B \cdot B_{max} = 2.866 \times 10^{-10} \cdot \Omega \cdot m$$

$$\rho_{och} := \frac{\rho_{RT}}{RRR_{st}} = 1.398 \times 10^{-10} \cdot \Omega \cdot m$$

$$\rho_{cBs} := \rho_{och} + m_B \cdot B_{max} = 2.586 \times 10^{-10} \cdot \Omega \cdot m$$

Average over copper resistivity for Cu Matrix and Cu Channel:

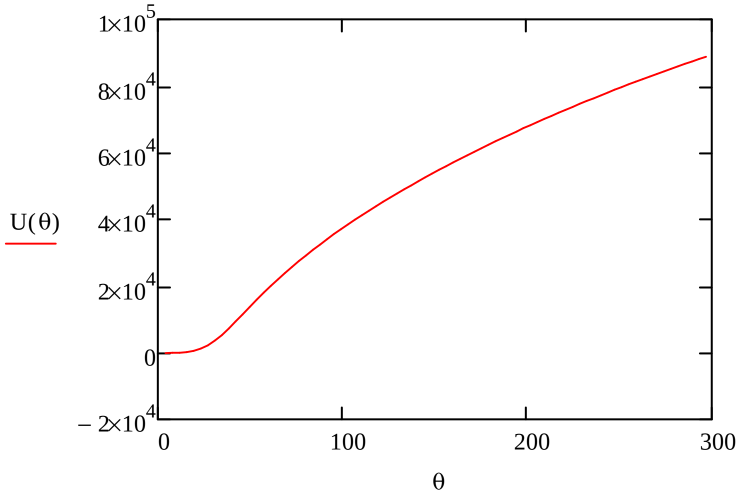
$$\rho_{cB} := \left(\frac{A_{st}}{\rho_{cBs}} + \frac{A_{wcu}}{\rho_{cBw}} \right)^{-1} \cdot A_{Cut} = 4.409 \times 10^{-10} \cdot \Omega \cdot m$$

U-Function using average winding density, average winding specific heat and average winding resistivity.

$$U(\theta) := \int_{\theta_0}^{\theta} \frac{\gamma_{av} \cdot C_{av}(\theta)}{\rho_{av}(\theta)} d\theta$$

Equation 9.4 from MW's book

$$\theta := 4.2 \cdot K, 8 \cdot K .. 300 \cdot K$$



To keep the peak temperature below 100 K, the decay time must be less than:

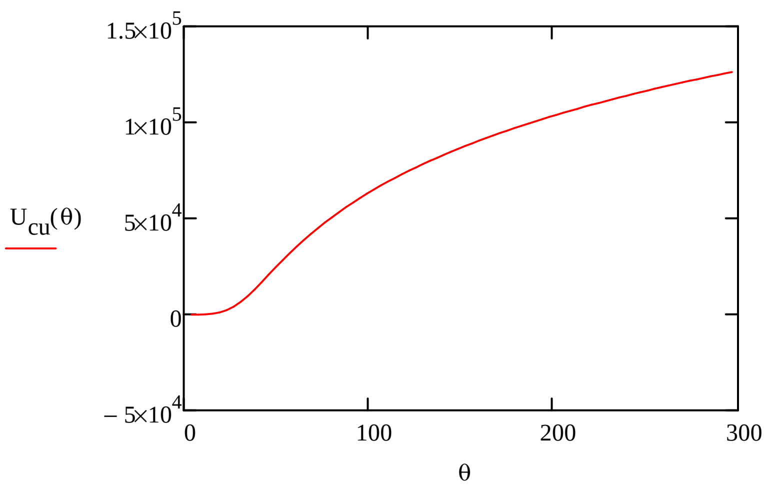
$$T_d := \frac{U(100 \cdot K)}{{J_0}^2} \qquad U(100 \cdot K) = 3.743 \times 10^{16} \cdot A^2 \cdot s \cdot m^{-4} \qquad T_d = 84.398\,s$$

U-Function using copper density, copper specific heat and copper resistivity.

$$U_{cu}(\theta) := \int_{\theta_0}^{\theta} \frac{\gamma_{Cu} \cdot C_{Cu}(\theta)}{\rho_{Cu}(\theta, 50)}\, d\theta$$

Equation 9.4 from MW's book

$\theta := 4.2 \cdot K, 8 \cdot K .. 300 \cdot K$



To keep the peak temperature below 100 K, the decay time must be less than:

$$T_{cud} := \frac{U_{cu}(100 \cdot K)}{{J_0}^2} \qquad U_{cu}(100 \cdot K) = 6.345 \times 10^{16} \cdot A^2 \cdot s \cdot m^{-4}$$

$T_{cud} = 143.081\,s$

Quench Velocities

$$V_z = \frac{J_0}{\gamma \cdot C} \cdot \left(\frac{L_0 \cdot \theta_s}{\theta_s - \theta_0} \right)^{\frac{1}{2}}$$

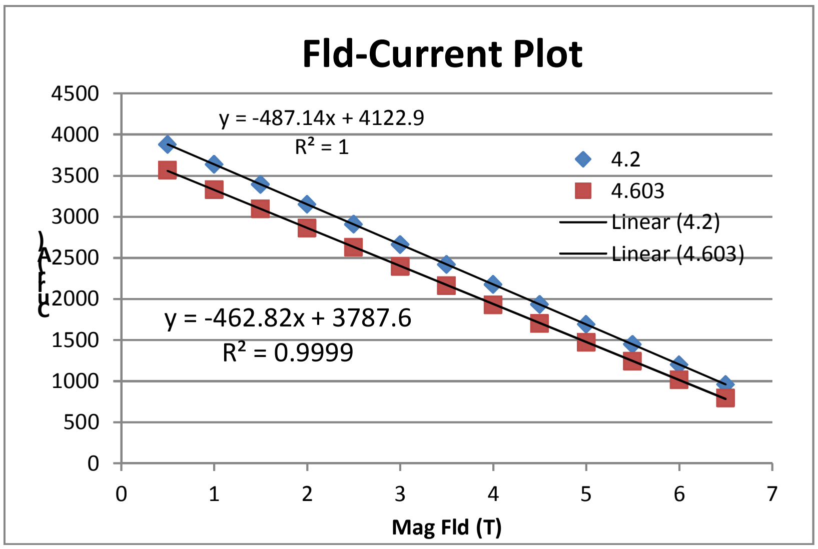
Equation 9.18 from MW's book

Mean Quench Temperature

θ_s is average of the generation temperature (when current sharing starts) and the critical temperature when the conductor is fully normal.

$$\left(I_c(B) := 3787.6\ A - 462.82 \cdot A \cdot T^{-1} \cdot B \right)$$

$$J_c := \frac{I_c(B_{max})}{A_{NbTi}} \qquad J_c = 1.825 \times 10^3 \cdot A \cdot mm^{-2} \qquad J_c \cdot A_{NbTi} = 2.413 \times 10^3\ A$$



$\theta_{c0} := 9.35 \cdot K$ Zero field critical temperature

$B_{c0} := 14.05 \cdot T$ Zero temperature critical field

$$J_{sc} := \frac{I_0}{A_{NbTi}} \qquad J_{sc} = 1.134 \times 10^3 \cdot A \cdot mm^{-2}$$ current density in the superconductor

$$\theta_c := \theta_{c0} \cdot \left(1 - \frac{B_{max}}{B_{c0}} \right)^{0.59} \qquad \theta_g := \theta_c - (\theta_c - \theta_0) \cdot \frac{J_{sc}}{J_c}$$

$$\theta_c = 8.128\,K \qquad \theta_g = 5.937\,K \qquad \left(\Delta\theta_{margin} := \theta_g - \theta_0 = 1.334\,K \right)$$

$$\theta_s := \frac{\theta_c + \theta_g}{2} \qquad \theta_s = 7.032\,K$$

Longitudinal Quench Velocity

$L_0 := 2.45 \cdot 10^{-8} W \cdot ohm \cdot K^{-2}$ Lorenz number

Adiabatic quench velocity for conductors which are not cooled, e.g., fully impregnated coil is given by equation 9.18 in MW's book

$$V_z := \frac{J_0}{\gamma_{av} \cdot C_{av}(\theta_s)} \cdot \left(\frac{L_0 \cdot \theta_s}{\theta_s - \theta_0} \right)^{\frac{1}{2}} \qquad V_z = 2.096 \cdot m \cdot s^{-1}$$

This estimate includes the specific heat capacity of the epoxy glass filler, however, MW argues that this should be excluded and the heat capacity of the metal only should be used:

$$C_m(\theta) := \frac{A_{CuCh} \cdot \gamma_{Cu} \cdot C_{Cu}(\theta) + A_{Cu} \cdot \gamma_{Cu} \cdot C_{Cu}(\theta) + A_{NbTi} \cdot \gamma_{NbTi} \cdot C_{NbTi}(\theta)}{A_{CuCh} \cdot \gamma_{Cu} + A_{Cu} \cdot \gamma_{Cu} + A_{NbTi} \cdot \gamma_{NbTi} + A_{vo} \cdot \gamma_{vo}}$$

$$C_m(\theta_s) = 0.511 \cdot J \cdot kg^{-1} \cdot K^{-1}$$

and we have:

$$V_{zm} := \frac{J_0}{\gamma_{av} \cdot C_m(\theta_s)} \cdot \left(\frac{L_0 \cdot \theta_s}{\theta_s - \theta_0} \right)^{\frac{1}{2}} \qquad V_{zm} = 1.918 \cdot m \cdot s^{-1}$$

Transverse Propagation

Thermal Conductivities

Use the conductivity at 4 K as representative of the range 1.8 K to 6.5 K scaled by the resistivity to account for magneto resistance.

Reference:J:\CLAS12 Magnet\Expe_Pers\Quench_RRG_PKG\Material data\Copper thermal conductivity_v11.xmcd

Reference:J:\CLAS12 Magnet\Expe_Pers\Quench_RRG_PKG\Material data\Epoxy Glass Thermal Conductivity.xmcd

$$k_{Cu}(\theta_0, 50, 3T) = 235.924 \cdot W \cdot m^{-1} \cdot K^{-1}$$

Magneto resistance scaling factor with RRR.

$$Cu_{mag100} := \frac{\rho_{Cu}(\theta_s, 100)}{\rho_{Cu}(\theta_s, 100) + \Delta\rho_{Cu}(B_{max})} \qquad Cu_{mag200} := \frac{\rho_{Cu}(\theta_s, 120)}{\rho_{Cu}(\theta_s, 120) + \Delta\rho_{Cu}(B_{max})}$$

$$Cu_{mag100} = 0.555 \qquad Cu_{mag200} = 0.509$$

$$k_{EG} := 0.05 \cdot W \cdot m^{-1} \cdot K^{-1}$$

$$k_{NbTi} := 1 \cdot W \cdot m^{-1} \cdot K^{-1}$$

$$k_{so} := 1 \cdot W \cdot m^{-1} \cdot K^{-1}$$

Thermal conductivity for NbTi and Solder needs to be verified!!!!

Longitudinal Conductivity with solder and epoxy

$$k_z := \frac{k_{Cu}(\theta_s, 120, B) \cdot A_{CuCh} \cdot Cu_{mag200} + k_{Cu}(\theta_s, 100, B) \cdot A_{Cu} \cdot Cu_{mag100} + k_{NbTi} \cdot A_{NbTi}}{A_c - A_{EG} - A_{vo}}$$

$$k_z = 361.556 \cdot W \cdot m^{-1} \cdot K^{-1}$$

Turn to Turn Conductivity

The conductivity across the conductor can be estimated as:

$$k_c := \frac{k_{Cu}(\theta_s, 120, B) \cdot A_{CuCh} \cdot Cu_{mag200} + k_{Cu}(\theta_s, 100, B) \cdot A_{Cu} \cdot Cu_{mag100} + k_{NbTi} \cdot A_{NbTi}}{A_{CuCh} + A_{Cu} + A_{NbTi} + A_{vo}}$$

$$k_c = 356.675 \cdot W \cdot m^{-1} \cdot K^{-1}$$

$$k_t := \left(\frac{t_{st}}{k_c} + \frac{t_u - t_{st}}{k_{EG}} \right)^{-1} \cdot t_u$$

$$k_t = 0.071 \cdot W \cdot m^{-1} \cdot K^{-1}$$

Turn to Turn Quench Velocity

$$V_{tt} := V_z \cdot \left(\frac{k_t}{k_z} \right)^{\frac{1}{2}} \qquad V_{tt} = 0.029 \cdot m \cdot s^{-1} \qquad \left(\frac{k_t}{k_z} \right)^{\frac{1}{2}} = 0.014$$

Layer to Layer Conductivity

$$k_l := \left(\frac{w_{st}}{k_c} + \frac{w_u - w_{st}}{k_{EG}} \right)^{-1} \cdot w_u \qquad k_l = 0.237 \cdot W \cdot m^{-1} \cdot K^{-1}$$

Layer to Layer Quench Velocity

$$V_{ll} := V_z \cdot \left(\frac{k_l}{k_z} \right)^{\frac{1}{2}} \qquad V_{ll} = 0.054 \cdot m \cdot s^{-1}$$

Propagation Times

Quenches are most likely to start on the coil bore where the magnetic field is highest. The time for the whole coil to become normal is then of the order of:

$$T_z := \frac{L_p}{2 \cdot V_z} \qquad T_z = 1.656\,s \qquad T_{tt} := \frac{W_T}{2V_{tt}} \qquad T_{tt} = 1.991\,s$$

$$T_{ll} := \frac{H_L}{V_{ll}} \qquad T_{ll} = 3.245\,s$$

(assumes the quench starts at the centre of the coil bore)

Minimum Propogating Zone

The radius of the minimum propogating zone is given by equation 5.10 in Wilson.

$\lambda_w := \lambda_{NbTi} + \lambda_{Cu} + \lambda_{CuCh} + \lambda_{vo}$ (fraction of the winding occupied by the conductor)

$$\lambda_w = 0.571 \qquad \lambda_{sc} := \lambda_{NbTi}$$

$$J_{sc} := \frac{I_0}{A_{NbTi}} \qquad J_{sc} = 1.134 \times 10^3 \cdot A \cdot mm^{-2} \qquad J_w := \frac{I_0}{A_c - A_{EG}} \qquad J_w = 36.906 \cdot A \cdot mm^{-2}$$

$$\rho_{op} := \frac{A_{CuCh} + A_{Cu}}{\dfrac{A_{CuCh}}{\rho_{CuB}(\theta_0, 120) + \Delta\rho_{Cu}(B_{max})} + \dfrac{A_{Cu}}{\rho_{CuB}(\theta_0, 100) + \Delta\rho_{Cu}(B_{max})}}$$

$\rho_{op} = 4.007 \times 10^{-10} \cdot ohm \cdot m$ (resistivity at the operating temperature)

$G_c := \dfrac{\rho_{op} \cdot \lambda_{sc}^2 J_{sc}^2}{1 - \lambda_{sc}} \qquad G_c = 1.811 \times 10^5 \cdot W \cdot m^{-3}$ (critical heat generation rate)

Radius of MPZ is given by equation 5.10 from MW's book

$$MPZ := \pi \cdot \left[\frac{k_z \cdot (\theta_c - \theta_g)}{\lambda_w \cdot G_c}\right]^{\frac{1}{2}} \qquad MPZ = 275.089 \cdot mm$$

Martin Wilson uses a different formula (Difficult to understand from where he take this formula) to calculate X_g (radius of MPZ), also instead of longitudianl thermal conductivity in his calculations he takes only copper thermal conductivity. Calculations for X_g from other Martin Wilson report shows slightly different values that is calculated here .

$$MPZ_{mw} := \left[\frac{2k_z \cdot (\theta_c - \theta_g)}{G_c}\right]^{\frac{1}{2}} \qquad MPZ_{mw} = 93.541 \cdot mm$$

Thermal conductivity over copper section from MW's

$$k_{cu} := \frac{L_0}{\rho_{cBw}} = 0.085 \frac{1}{mm \cdot K^2} \cdot W \qquad k_{con} := \lambda_{cu} \cdot \frac{L_0}{\rho_{cBw}} = 0.047 \frac{1}{mm \cdot K^2} \cdot W$$

$$X_g := \sqrt{2 \cdot k_{con} \cdot \theta_s \cdot \frac{(\theta_c - \theta_g)}{G_c}} \qquad X_g = 88.987\,mm$$

$\beta := \dfrac{\theta_g - \theta_0}{\theta_0}$ $\beta = 0.29$ $a := 1.1, 1.2 .. 4.3$ a is same as "m" in MW's book

$\nu(a) := \dfrac{\pi \cdot (1+\beta) \cdot (a-1)}{\beta \cdot a}$ Equation 5.22 Wilson

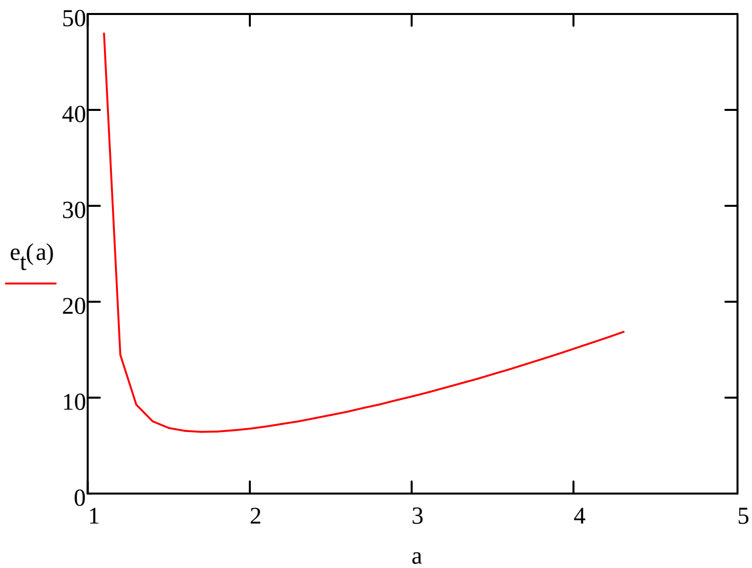
$e_g(a) := \left[\dfrac{a \cdot \beta}{\pi \cdot (a-1)}\right]^4 \cdot \left(\nu(a)^4 + 3.8 \cdot \nu(a)^3 + 9 \cdot \nu(a)^2 + 11.6 \cdot \nu(a) + 6.3\right) - 1$ Equation 5.21 Wilson

$\eta(a) := \dfrac{a - 1 - \beta}{a \cdot \beta}$ Equation 5.25 Wilson, This equation has a typo in the book, checked with MW on March 9th 2013.

$$e_h(a) := \left(\dfrac{a \cdot \beta}{a-1}\right)^4 \cdot \left[\left(3 - \dfrac{3}{a}\right) + 12 \cdot \eta(a) \cdot \ln(a) + 18 \cdot \eta(a)^2 \cdot (a-1) + 6 \cdot \eta(a)^3 \cdot \left(a^2 - 1\right) + \eta(a)^4 \cdot \left(a^3 - 1\right)\right] - \left(a^3 - 1\right)$$

Equation 5.24 Wilson

$e_t(a) := e_g(a) + e_h(a)$



The energy iput needed to establish a propagating zone is e_t=e_h+e_g , the outstanding variable is "a". For a fixed value of ß the energy e_t passes through a broad minimum with repsect to "a". We take the minimum value of e_t as the best estimate of "a".

Guess a value of "a" for minimum value of e_t

$a := 2.5$

Given $a > 1$ $a_{min} := \text{Minimize}\left(e_t, a\right) = 1.713$

Estimate specific enthalpies from specific heat capacities:

$\gamma H_{Cu} := \left(\dfrac{L_{Cu} \cdot \theta_0^{\,4}}{4} + \dfrac{\gamma_{Cue} \cdot \theta_0^{\,2}}{2}\right) \cdot \gamma_{Cu}$ $\gamma H_{Cu} = 2.297 \times 10^3 \cdot J \cdot m^{-3}$

$\gamma H_{NbTi} := 4.5 \cdot 10^3 \cdot J \cdot m^{-3} \cdot \left(\frac{\theta_0}{4 \cdot K}\right)^4$ $\gamma H_{NbTi} = 7.891 \times 10^3 \cdot J \cdot m^{-3}$

$\gamma H_{ins} := \left(\frac{L_{eg} \cdot \theta_0^4}{4}\right) \cdot \gamma_{EG}$ $\gamma H_{ins} = 99.969 \cdot J \cdot m^{-3}$

$\gamma H_0 := \lambda_{cu} \cdot \gamma H_{Cu} + \lambda_{NbTi} \cdot \gamma H_{NbTi} + \lambda_{ins} \cdot \gamma H_{ins}$ $\gamma H_0 = 1.44 \times 10^3 \cdot J \cdot m^{-3}$

$\alpha := \sqrt{\frac{V_{ll} \cdot V_{tt}}{V_z^2}}$ $\alpha = 0.019$ $\alpha \cdot MPZ = 5.213 \cdot mm$

$R_g := MPZ$ $E_0 := \frac{4 \cdot \pi}{3} \cdot \alpha^2 \cdot R_g^3 \cdot \gamma H_0$ Equation 5.20 from MW's book

$E_0 = 0.045 \cdot J$ 1.7 is the value of "a" for minimum e_t

$E_{tot} := e_g(a_{min}) \cdot E_0$

$E_{tot} = 0.202 \cdot J$ energy required E_{tot} to heat up the generation region MPZ

Energy density: $e_{tot} := \frac{E_{tot}}{\alpha \cdot MPZ^2}$ $e_{tot} = 1.41 \times 10^{-4} \cdot J \cdot mm^{-2}$

This energy may be unrealistic if the width of the MPZ is less than the conductor width as in practice it is likely that the whole conductor cross section would be at the same temperature. We can address this either by calculating the energy required to send a MPZ length of conductor normal or by calculating the length of conductor giving a volume equal to the MPZ:

$E_{MPZ} := e_g(a_{min}) \cdot A_c \cdot R_g \cdot \gamma H_0$ $E_{MPZ} = 0.127 \cdot J$

$L_{MPZ} := \frac{\frac{4 \cdot \pi}{3} \cdot \alpha^2 \cdot R_g^3}{A_c}$ $L_{MPZ} = 439.537 \cdot mm$

Estimate the amount of movement required from a length of wire equal to the MPZ in order to cause a quench:

$\frac{E_{tot}}{B_{max} \cdot I_0 \cdot MPZ} = 0.165 \cdot mm$ or: $\frac{E_{MPZ}}{B_{max} \cdot I_0 \cdot MPZ} = 0.103\,mm$

Peak Temperature

Guess a peak temperature: $\theta_p := 44.9 \cdot K$ adjust until the estimated temperature agrees at θ_{maxest}

$T_Q := \left(\frac{90 \cdot L_{tot} \cdot U(\theta_p)^2 \cdot A_c^2}{4 \cdot \pi \cdot J_0^4 \cdot \rho_{av}(\theta_p) \cdot \alpha^2 \cdot V_z^3}\right)^{\frac{1}{6}}$ $T_Q = 24.782\,s$

Actual peak temperatures can be calculated from:

$$\theta_{maxest} := \frac{J_0^{\,4} \cdot T_Q^{\,2} \cdot \theta_p}{U(\theta_p)^2}$$

$\theta_{maxest} = 44.894 \cdot K$

Boundary Effects

Boundary effects are likely to be important as the normal zone stops expanding when boundaries are reached, hence the decay time is prolonged. Boundary encounter times are:

$T_z = 1.656\,s$ $\quad T_{tt} = 1.991\,s$ $\quad T_{ll} = 3.245\,s$

Guess a peak temperature: $\theta_{p3d} := 182.4K$ adjust until the estimated temperature agrees with θ_{max}

$$T_{Q3d} := \left(\frac{90 \cdot L_{tot} \cdot U(\theta_{p3d})^2 \cdot A_c^{\,2}}{4 \cdot \pi \cdot J_0^{\,4} \cdot \rho_{av}(\theta_{p3d}) \cdot \alpha^2 \cdot V_z^{\,3}} \right)^{\frac{1}{6}}$$

$T_{Q3d} = 29.237\,s$

Taking into account boundary encounters we can calculate a dimensionless decay time:

$$t_{d1} := \left(3 \cdot \frac{T_z}{T_{Q3d}} \right)^{\frac{-1}{5}} \qquad t_{d2} := \left(\frac{15}{2} \cdot \frac{T_z}{T_{Q3d}} \cdot \frac{T_{tt}}{T_{Q3d}} \right)^{\frac{-1}{4}} \qquad t_{d3} := \left(20 \cdot \frac{T_z}{T_{Q3d}} \cdot \frac{T_{tt}}{T_{Q3d}} \cdot \frac{T_{ll}}{T_{Q3d}} \right)^{\frac{-1}{3}}$$

(note td1 & td2 may have different arguments depending on the relative times)

$t_d := t_{d3}$ as appropriate.

$t_d = 4.888$ $\qquad t_d \cdot T_{Q3d} = 142.911\,s$

The peak temperature is now:

$$\theta_{max} := \frac{J_0^{\,4} \cdot t_d^{\,2} \cdot T_{Q3d}^{\,2} \cdot \theta_{p3d}}{U(\theta_{p3d})^2}$$

$\theta_{max} = 182.402 \cdot K$

Peak internal voltage:

$$V_{max} := \frac{2.5 \cdot L_{tot} \cdot I_0}{T_{Q3d}} \cdot \left(\frac{T_z}{T_{Q3d}} \cdot \frac{T_{tt}}{T_{Q3d}} \cdot \frac{T_{ll}}{T_{Q3d}} \right)^{\frac{1}{3}}$$

$V_{max} = 277.538 \cdot volt$

$$L_{tot} \cdot \frac{I_0}{t_d \cdot T_{Q3d}} = 301.341\,V$$

$V_{tot} := L_{tot} \cdot \frac{I_0}{t_d \cdot T_{Q3d}}$ $V_{tot} = 301.341\,V$ $dV := V_{tot} - V_{max}$

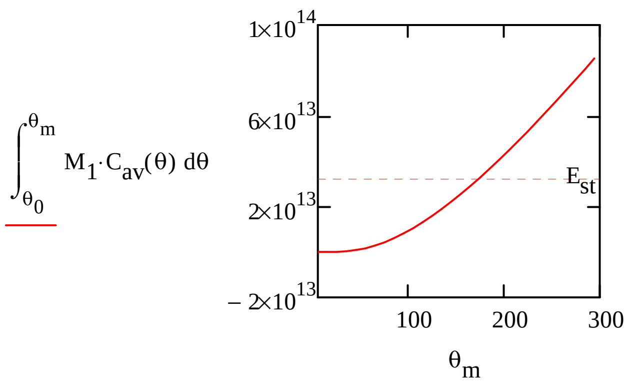
$dV = 23.804\,V$

Mean Temperature

Coil mass: $M_1 := H_L \cdot W_T \cdot L_p \cdot \gamma_{av}$ $M_1 = 809.73\,kg$

Mean temperature assuming the coil absorbs the stored energy evenly:

$\theta_m := 4.4 \cdot K, 14.4 \cdot K .. 300 \cdot K$



$\theta_{mean} := 80 \cdot K$ $\theta_0 = 4.603\,K$

Given

$$E_{st} = \int_{\theta_0}^{\theta_{mean}} M_1 \cdot C_{av}(\theta)\, d\theta$$ $E_{st} = 3.23 \times 10^7\,J$

$\theta_{meanerr} := \text{Minerr}(\theta_{mean})$ $\theta_{meanerr} = 173.048\,K$

In the time for the quench to fill the whole coil, what is the temperature rise at the initiation point?

$\Delta\theta := 80 \cdot K$

Given $T_{ll} \cdot J_0^2 = U(\Delta\theta)$

$\Delta\theta_{err} := \text{Minerr}(\Delta\theta)$ $\Delta\theta_{err} = 23.747\,K$

Coil resistance: $R_{coil} := \frac{\rho_{av}(\Delta\theta_{err})}{A_{Cu} + A_{NbTi}} \cdot L_p \cdot N_T \cdot N_L$ $R_{coil} = 0.118\,\Omega$

Coil voltage: $R_{coil} \cdot I_0 = 177.228\,V$