4.2 Time-Walk Correction

The next step involved examining the ADC vs. TDC two-dimensional spectra for PMT1 and PMT2. A slide bend of the locus at low values of the ADC indicates that a time-walk correction [5] needs to be applied to the data, despite the use of Tennelec TC453 constant fraction discriminators (CFDs) that overcome this problem electronically. This arises from the time variation associated with the signals having different pulse heights when crossing the discriminator threshold. The slight bend possibly indicates that CFDs were not set optimally. The walk correction for PMT i can be written as:

$$t_{ic} = t_i - \frac{w_i}{\sqrt{q_i}},\ i = 1, 2 \tag{3}$$

where q_i, t_i and t_{ic} are the measured integrated charge of the PMT's signal (i.e. the pedestal-corrected ADC value), the measured TDC time, the time-walk corrected TDC time, and w_i is a parameter obtained form fitting this equation to the ADC vs. TDC locus.

The resulting t_{ic} distributions can be fitted using a Gaussian distribution and the extracted mean, μ_{ic}, and root mean square, σ_{ic}, can be used to re-check the quality of the data and to extract a more refined v_{eff}. In some cases, a Landau or Lorentz distribution yields more accurate results if the t_{ic} distributions possess a tail at large TDC values. Specifically, the plot of t_{ic} vs. the z-position should yield a straight line:

$$t_{ic} = (1/v_{eff})z + t_{io},\ i = 1, 2 \tag{4}$$

where t_{io} represents the offset (y-intercept) for that particular TDC. Finally, the σ_{ic} should be plotted versus z-position and compared to Figure 4 in reference [6].

4.3 Time of Hit

The time of hit (or particle arrival time) can be calculated next, based on t_{ic} and the hit position information. The hit position is determined from the approximate center of the upper trigger counter, as marked on Module 1 during the data collection. The relevant equation array follows:

$$t_{ic} = t_{ih} + t_{io} - \left(\frac{L}{2} + (-1)^i z\right) \Big/ v_{eff},\ i = 1, 2 \tag{5}$$

$$t_{arr} = t_{ih} = t_{ic} - t_{io} - \frac{L}{2v_{eff}}, \; i = 1, 2 \tag{6}$$

$$t_{arr} = \frac{t_{1c} + t_{2c}}{2} - \frac{t_{1o} + t_{2o}}{2} - \frac{L}{2v_{eff}} \tag{7}$$

where t_{ic}, t_{ih}/t_{arr}, t_{ref} are the time-walk corrected, hit/arrival and offset (reference) times, respectively, and z represents the hit position of the Module with $z = 0$ at its center (L is the length of Module 1: 400 cm). The reference time can be obtained from the TDC common start time [4]. As extracted in Equation (6), t_{ih} represents the hit timing as extracted independently from each PMT. The arrival time for Module 1 is better obtained using the weighted average from both its ends, via:

$$t_{av} = \overline{t_{ic}(z)} = \sum_{i=1}^{n} \left(\frac{t_{ih}(z)}{\sigma_{ih}^2(z)} \right) \Big/ \sum_{i=1}^{n} \left(\frac{1}{\sigma_{ih}^2(z)} \right), \; n = 1, 2 \tag{8}$$

The resulting t_{av} distributions can be fitted using a Gaussian distribution and the extracted σ_{av} should be plotted versus z-position and compared to Figure 4 in reference [6]. A fit, as explained in that reference, should be attempted.

The z-coordinate of the hit position is related to the time difference between the TDC values at the two ends of Module 1:

$$z = \frac{v_{eff}}{2}((t_{1c} - t_{2c}) - (t_{1o} - t_{2o})) \tag{9}$$

The center of this distribution determines $t_{1o} - t_{2o}$ and the width determines v_{eff}. This can be clearly seen from a plot of $t_{1c} - t_{2c}$ vs. the z-position. The extracted value of $t_{1o} - t_{2o}$ should agree with the difference of the two terms as extracted from Equation (4). The same should apply to the case of adding the two corrected TDCs, as is done when extracting the Mean Timer (MT). In fact, μ_{MT} and σ_{MT} should be extracted and plotted versus the z-position, as this is an additional quality check of the data and will be used in the extraction of the systematic error. For more details the reader is directed to reference [4].

4.4 Unfolding the Resolution

The following equation shows all contributions to the timing resolution:

$$\sigma_{av}^2 = \sigma_{ic}^2 + \sigma_{pos}^2 + \sigma_{off}^2 \tag{10}$$

where the labels follow the convention above and σ_{pos} represents the timing variation die to the finite size of the upped trigger counter. Specifically, the width, x, perpendicular to the $\hat{z}$-axis on the horizontal plane was 1" (2.54 cm) and therefore $\sigma_{pos} = (x/v_{eff})/\sqrt{12} = 47\ ps$ from the statistics of a uniform distribution. On the other hand, σ_{off} represents the width of the second term on the right hand side of Equation (7), is related to σ_{MT}, and can be extracted as explained above.

Additional cross checks can be carried out, by examining the TDC distribution of PMT3. The trigger counter resolution can be obtained trivially, if one possesses two identical counters with identical PMTs set to the same gain. Placing these counters next to each other and in coincidence results in:

$$\left(\sigma\left(\frac{t_{1c}+t_{2c}}{2}\right)\right)^2 = \left(\frac{\sigma_1}{2}\right)^2 + \left(\frac{\sigma_2}{2}\right)^2 = \left(\frac{\sigma_1}{\sqrt{2}}\right)^2 \tag{11}$$

In this cosmics experiment no such provision exists, so that the contribution to the resolution of the reference counter can be estimated using linear combination of the above equations. For example, Equation (11) is also applicable, in principle, at $z = 0$, where σ_1 and σ_2 can be assumed to be the same.

5 And the ADC?

Three equations to keep in mind are:

$$\sigma_{tot} \propto \frac{1}{\sqrt{N_{pe}}} \tag{12}$$

$$\sigma_{tot} = \sqrt{\frac{\tau^2 + TTS^2}{N_{pe}}} \tag{13}$$

$$N_{pe} = \left(\frac{\mu_{ADC}}{\sigma_{ADC}}\right)^2 \tag{14}$$

where σ_{tot}, N_{pe}, τ, TTS refer to the total experimental resolution, teh number of extracted photo-electrons, the decay time of the scintillator (from manufacturer's specifications, 3.2 ns for BCF-12 fibers) and the time transit spread of each PMT. The first two equations are related by the proportionality constant. Finally, μ_{ADC} represents the mean of the pedestal-subtracted ADC spectrum and σ_{ADC} is the rms of the ADC distribution.

Form the Geometric Mean (GM) of the ADCs from each cell:

$$GM = \sqrt{ADC_N \times ADC_S} \tag{15}$$