Recent Results from JPAC

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Workshop on Photoproduction
JLab, September 2019

Jefferson Lab
Thomas Jefferson National Accelerator Facility

JPAC Joint Physics Analysis Center

The Factorization Hypothesis

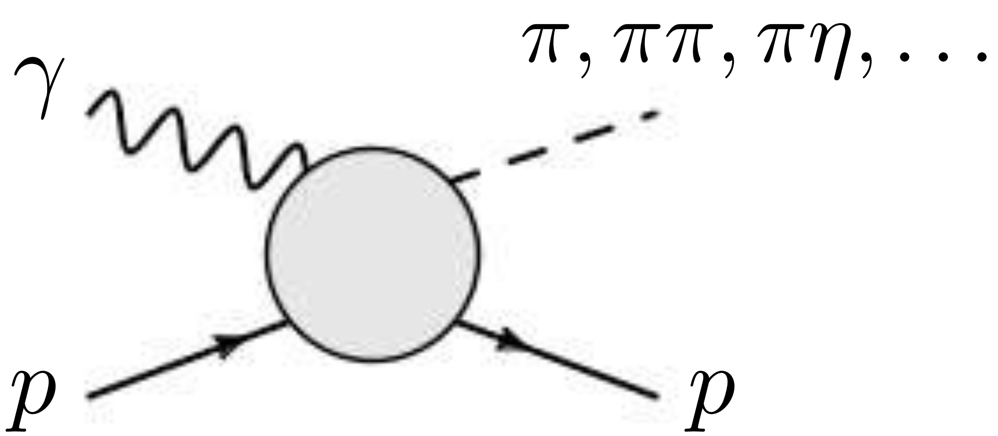
Photoproduction of mesons at $E_\gamma = 6 - 12\ \mathrm{GeV}$

Study photoproduction of mesons

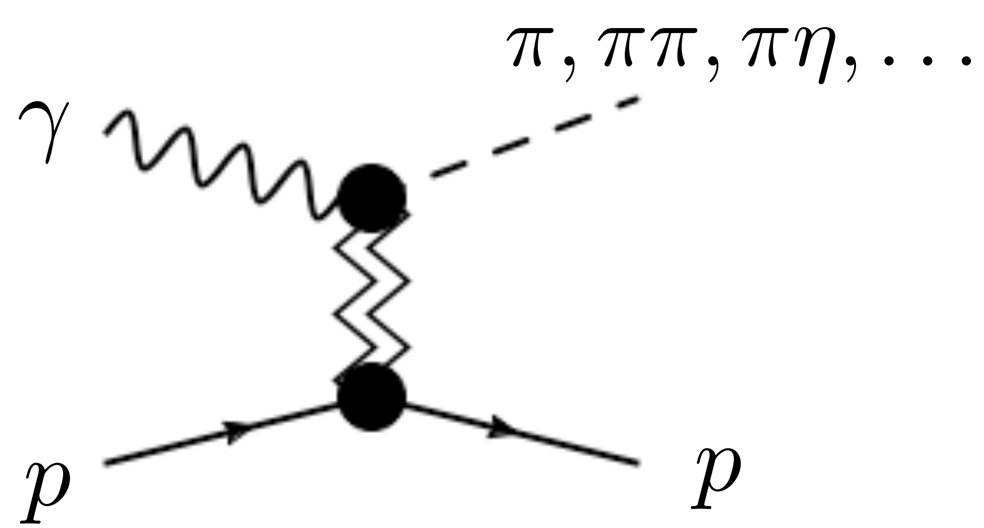
Search for exotic resonances

Special interest in mesons:

Does the target decouple at JLab energies ?



Factorization ?



JPAC publications on JLab physics

Single Meson Photoproduction:

$\vec{\gamma} p \to \pi N$ Mathieu et al **PRD92 074013 (2015)**
Mathieu et al **PRD98 014041 (2018)**

$\vec{\gamma} p \to \pi \Delta$ Nys et al **PLB779 77 (2018)**

$\vec{\gamma} p \to \eta p$ Nys et al **PRD95 034014 (2017)**
Mathieu et al **EPL122 41001 (2018)**

$\vec{\gamma} p \to \eta' p$ Mathieu et al **PLB774 362 (2017)**

Vector Meson Photoproduction:

$\vec{\gamma} p \to \rho^0 p$
$\vec{\gamma} p \to \omega p$
$\vec{\gamma} p \to \phi p$
} Mathieu et al **PRD97 094003 (2018)**

$\vec{\gamma} p \to J/\psi p$ Hiller Blin et al **PRD94 034002 (2016)**
Winney et al **PRD100 034019 (2019)**

Double Mesons Photoproduction:

$\vec{\gamma} p \to \pi^0 \eta p$ Mathieu et al arXiv:1906.04841
(to appear in PRD)

Inclusive Electroproduction:

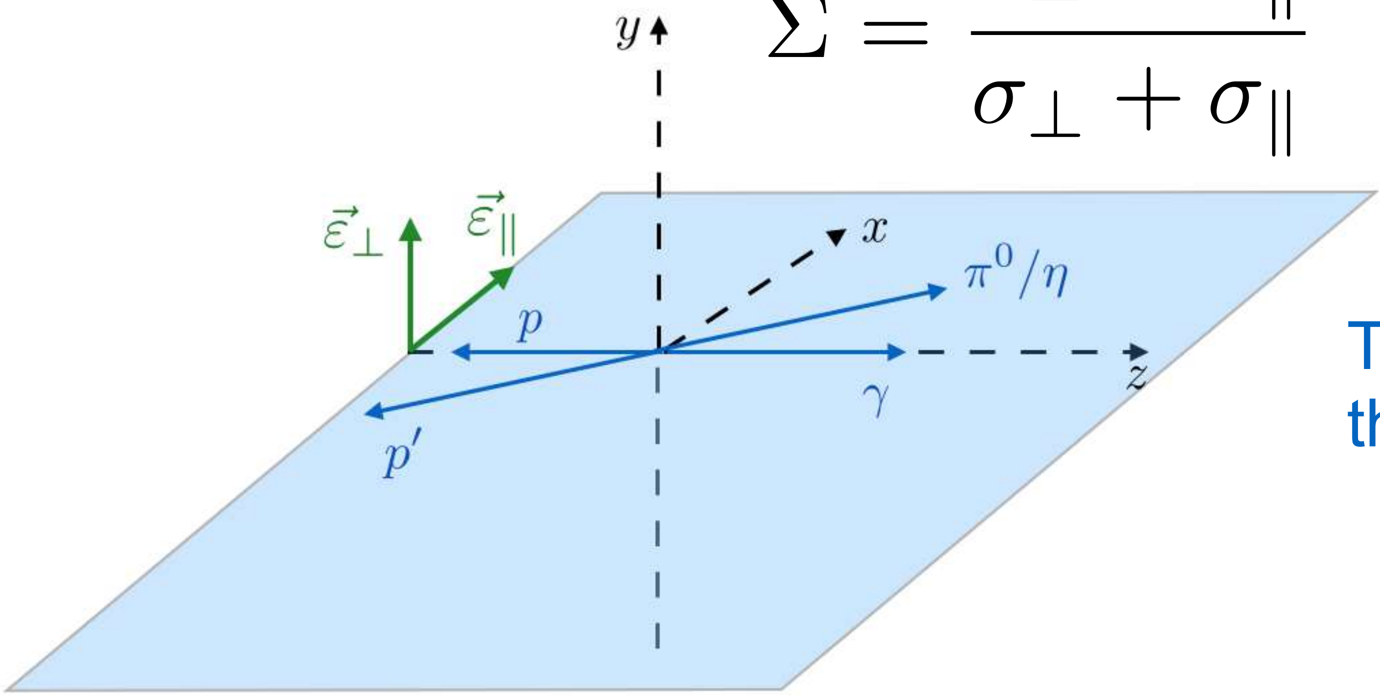
$e^- p \to e^- X$ Hiller Blin et al **PRC100 035201 (2019)**

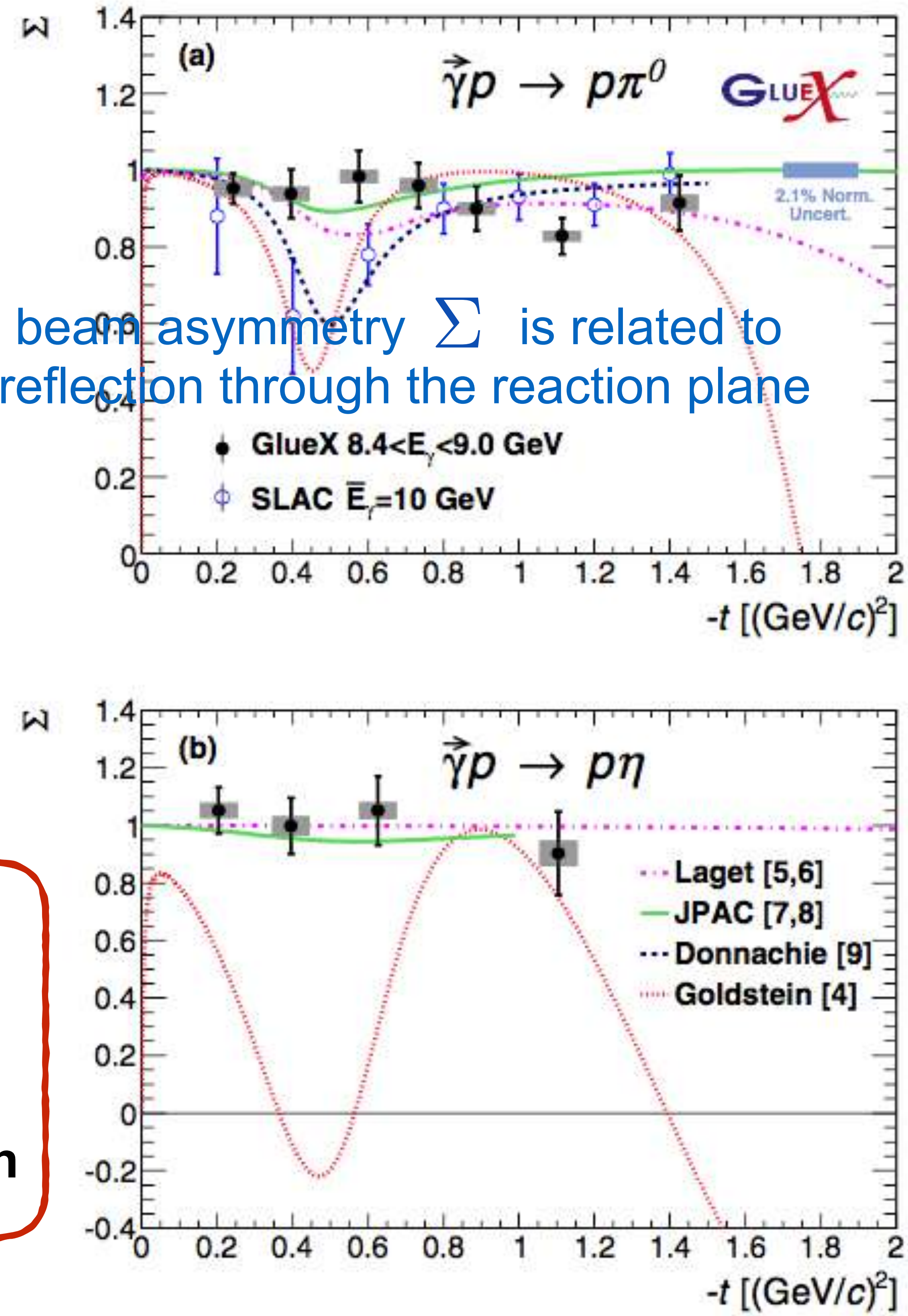
Simulations and codes available:

http://www.indiana.edu/~jpac/

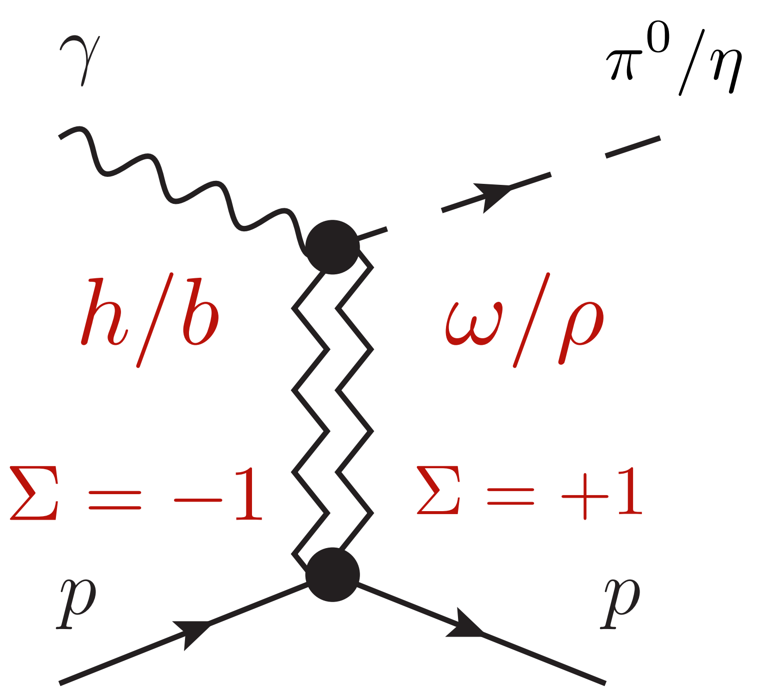
Single Meson Photoproduction

$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel}$$





The beam asymmetry Σ is related to the reflection through the reaction plane



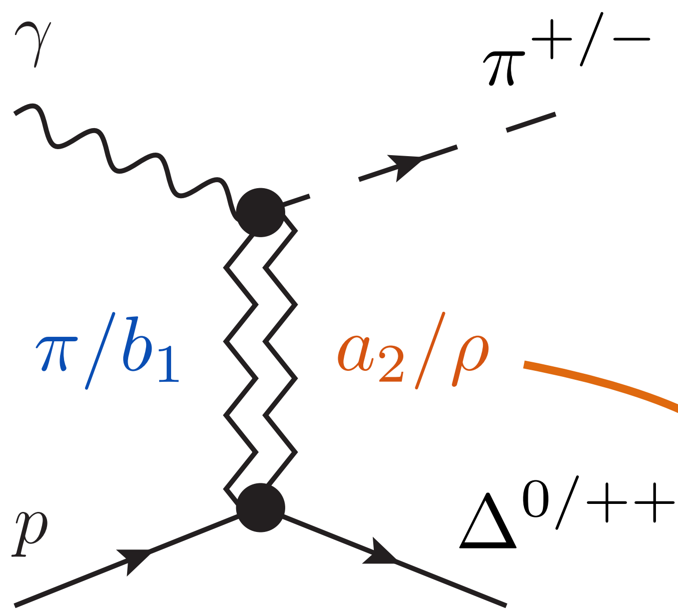
Dominance of vector meson exchange in both π^0/η photoproduction

GlueX, VM and Nys PRC95 (2017)

Single Meson Photoproduction

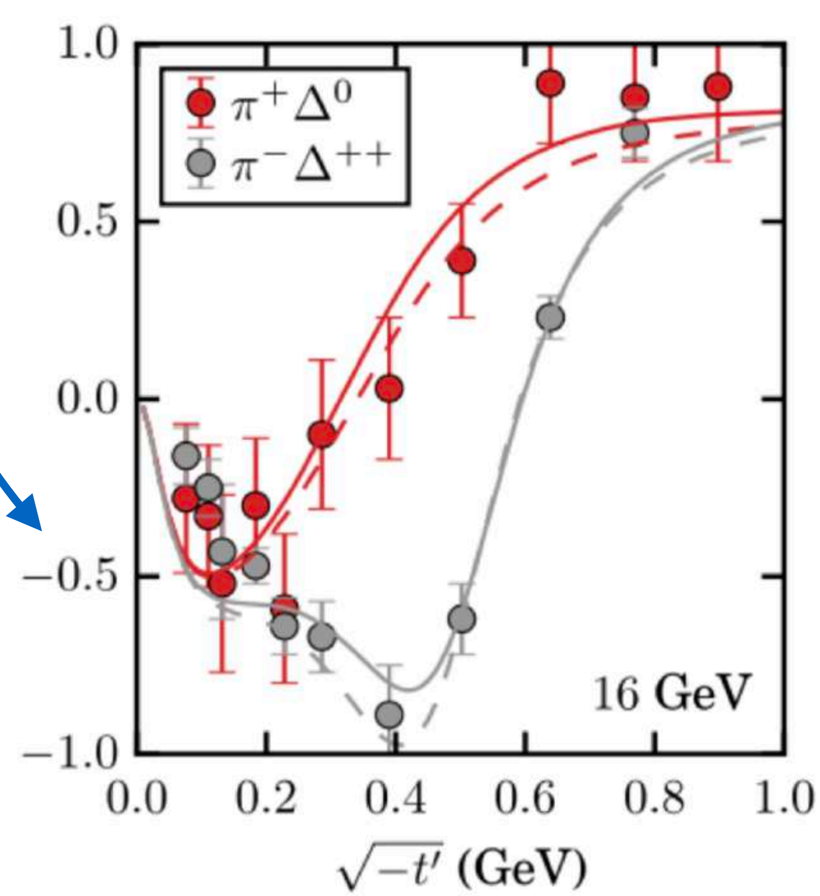
$$\sqrt{3}A(\gamma p \to \pi^+\Delta^0) = A^{\rho+b_1} + A^{a_2+\pi}$$

$$A(\gamma p \to \pi^-\Delta^{++}) = A^{\rho+b_1} - A^{a_2+\pi}$$

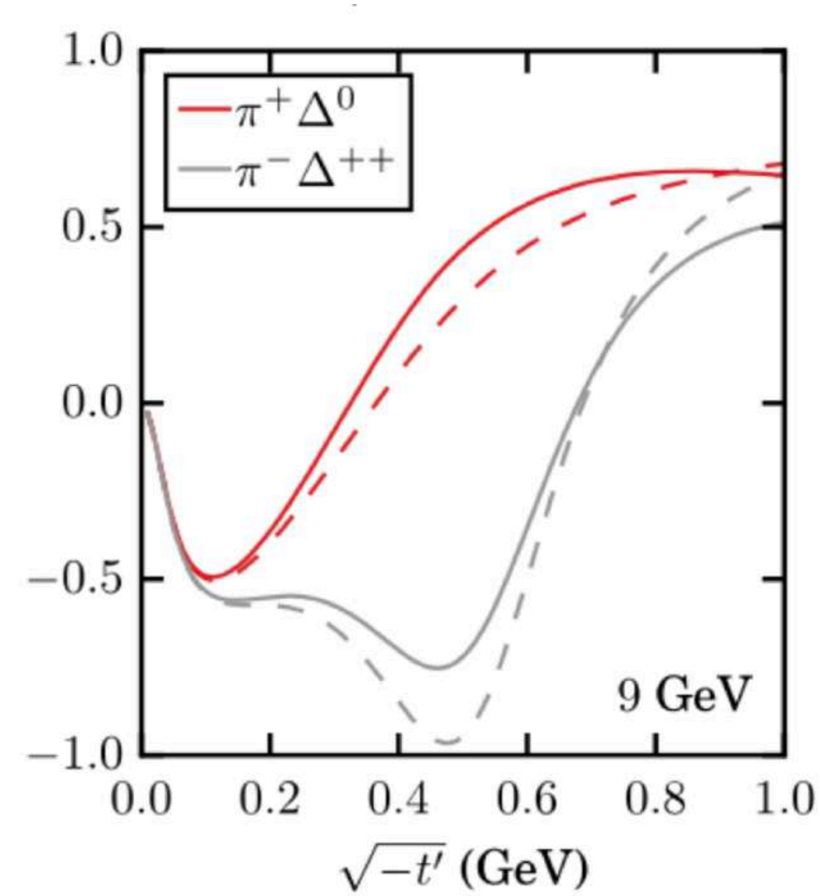


Dominance of natural exchanges at large |t|

Dominance of pion exchange at low |t|

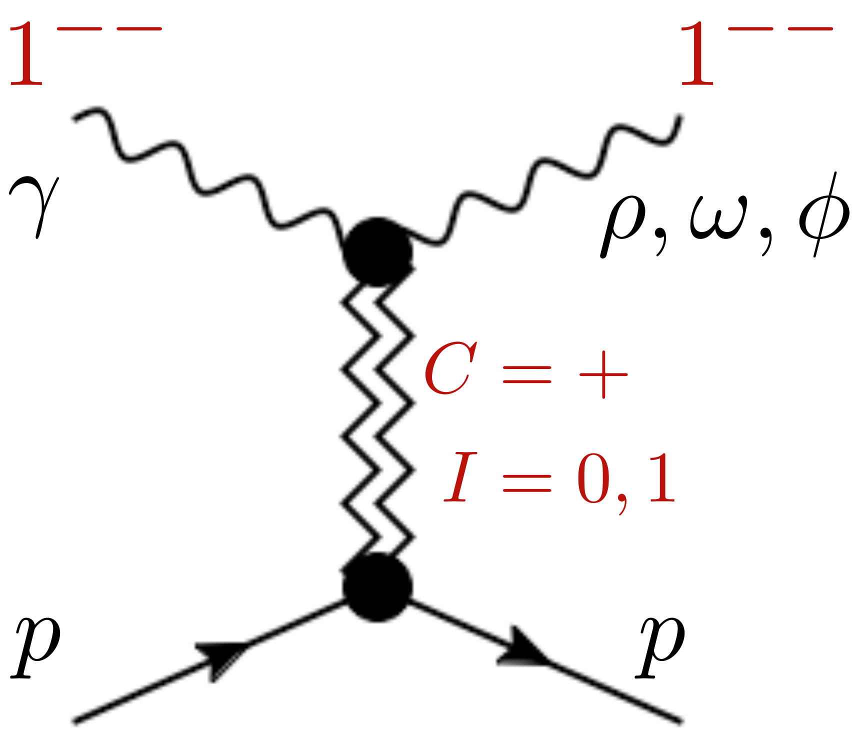


Prediction for GlueX



Nys et al **PLB779 77 (2018)**

Vector Meson Photoproduction



Probe different exchanges by combined analysis of ρ, ω, ϕ

Pomeron dominates at high energies

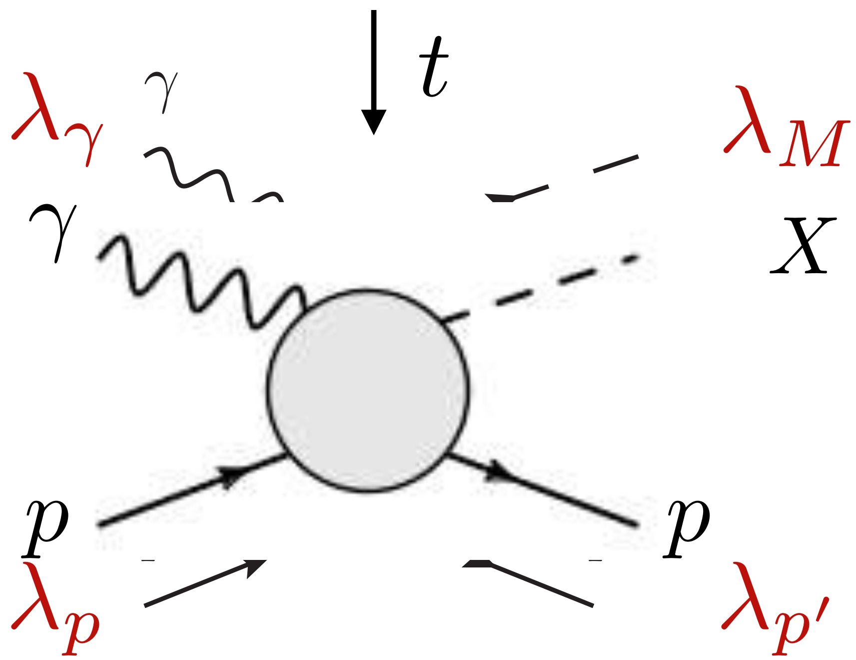
Use the angular distribution of the vector to extract spin density matrix elements

$$\frac{8\pi}{3}\frac{d\sigma}{d\Omega} = 1 - \rho^0_{00} + \left(3\rho^0_{00} - 1\right)\cos^2\theta - 2\sqrt{2}\mathrm{Re}\ \rho^0_{10}\sin 2\theta\cos\phi - 2\rho^0_{1-1}\sin^2\theta\cos 2\phi$$

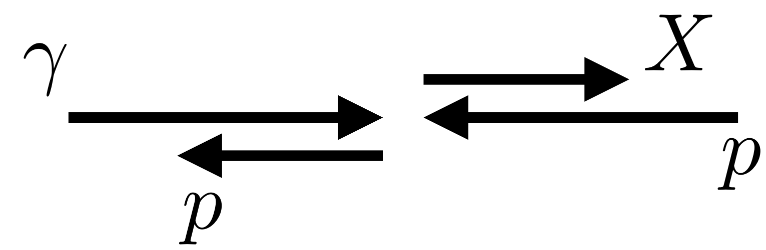
9 SDME accessible with linearly polarized beam

ρ^0_{00}	$\mathrm{Re}\ \rho^0_{10}$	ρ^0_{1-1}
ρ^1_{11}	$\mathrm{Re}\ \rho^1_{10}$	ρ^0_{11}
ρ^1_{1-1}	$\mathrm{Im}\ \rho^2_{10}$	$\mathrm{Im}\ \rho^2_{1-1}$

Factorization



Angular mom. conservation in forward direction:



$$A^{\lambda_\gamma \lambda_M}_{\lambda_p \lambda_{p'}} \propto \underbrace{(\sin\theta/2)}_{\sqrt{-t}}{}^{|\lambda_\gamma - \lambda_p + \lambda_{p'} - \lambda_M|}$$

Leading order in the energy :

$$A^{\lambda_\gamma \lambda_M}_{\lambda_p \lambda_{p'}} \propto \gamma(t)(\sqrt{-t})^{|(\lambda_\gamma - \lambda_M) - (\lambda_p - \lambda_{p'})|}$$

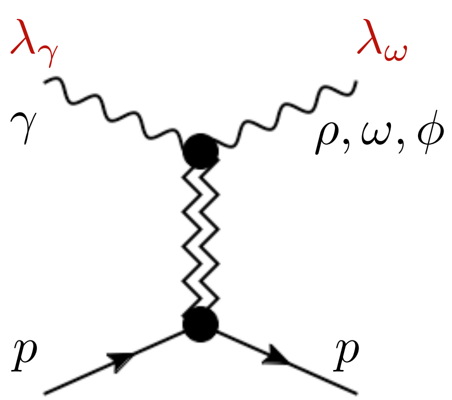
Factorization implies angular mom, conservation at each vertex:

$$A^{\lambda_\gamma \lambda_M}_{\lambda_p \lambda_{p'}} \propto \underbrace{\gamma(t)(\sqrt{-t})^{|\lambda_\gamma - \lambda_M|}}_{\text{top vertex}} \times \underbrace{(\sqrt{-t})^{|\lambda_p - \lambda_{p'}|}}_{\text{bottom vertex}}$$

Spin Density Matrix Elements

Use the angular distribution of the vector
to extract spin density matrix elements

$$\frac{8\pi}{3}\frac{d\sigma}{d\Omega} = 1 - \rho^0_{00} + \left(3\rho^0_{00} - 1\right)\cos^2\theta - 2\sqrt{2}\mathrm{Re}\ \rho^0_{10}\sin 2\theta\cos\phi - 2\rho^0_{1-1}\sin^2\theta\cos 2\phi$$



Structure at the top vertex:

$$T_{\lambda_\gamma\lambda_\omega} = \beta_0\left(\delta^{\lambda_\omega}_{\lambda_\gamma} + \beta_1\frac{\sqrt{-t}}{m_\omega}\delta^{\lambda_\omega}_0 + \beta_2\frac{-t}{m^2_\omega}\delta^{\lambda_\omega}_{-\lambda_\gamma}\right)$$

$$\rho^0_{00} = \frac{2}{N}\sum_{\lambda,\lambda'}\left|T_{\substack{1,0\\\lambda,\lambda'}}\right|^2$$

$$\mathrm{Re}\ \rho^0_{10} = \frac{1}{N}\mathrm{Re}\sum_{\lambda,\lambda'}\left(T_{\substack{1,1\\\lambda,\lambda'}} - T_{\substack{1,-1\\\lambda,\lambda'}}\right)T^*_{\substack{1,0\\\lambda,\lambda'}}$$

$$\rho^0_{1-1} = \frac{2}{N}\mathrm{Re}\sum_{\lambda\lambda'}T_{\substack{1,1\\\lambda,\lambda'}}T^*_{\substack{1,-1\\\lambda,\lambda'}}$$

→

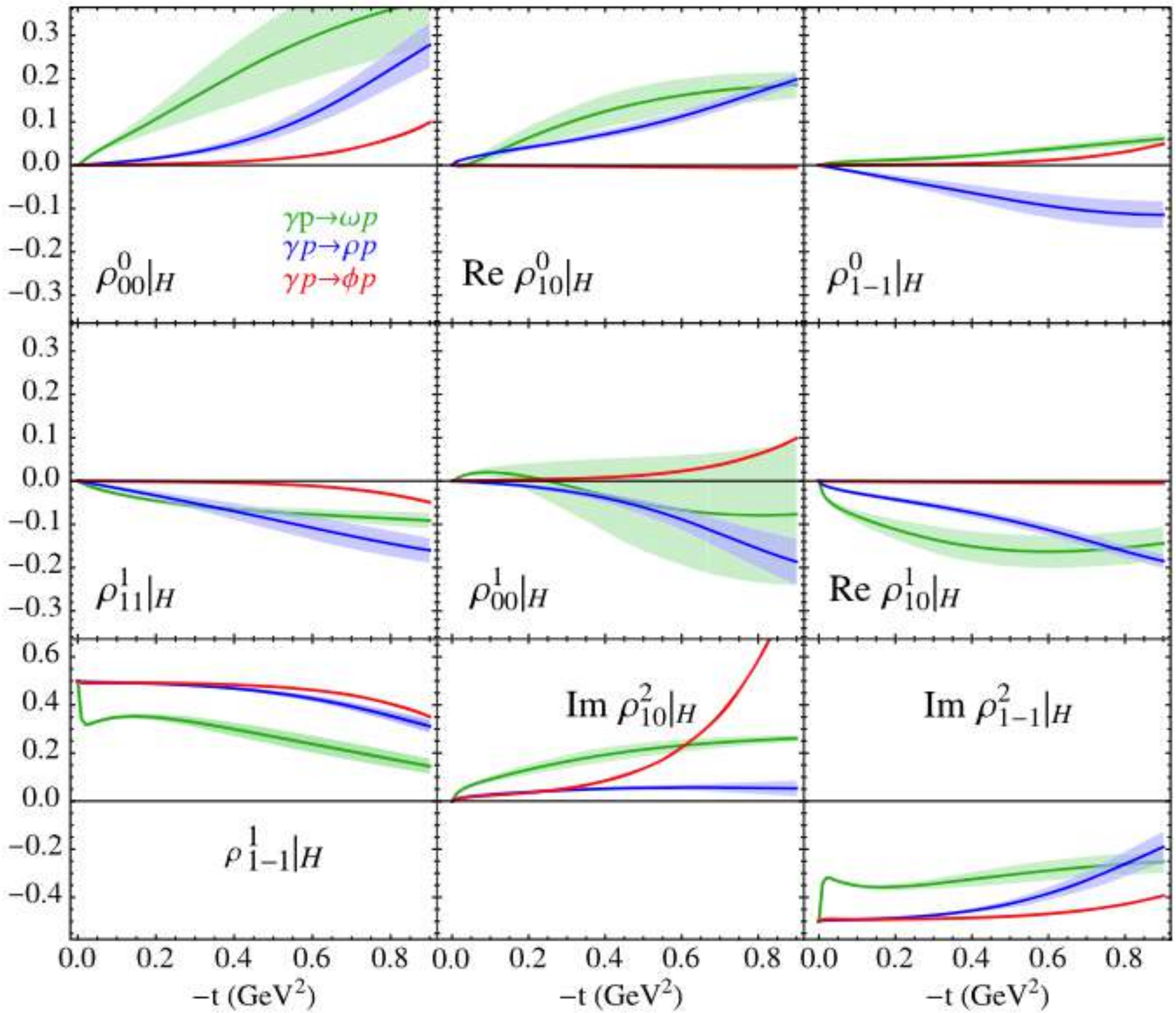
$$\rho^0_{00} \propto \beta_1^2\frac{-t}{m^2_\omega}$$

$$\mathrm{Re}\ \rho^0_{10} \propto \frac{1}{2}\beta_1\frac{\sqrt{-t}}{m_\omega}$$

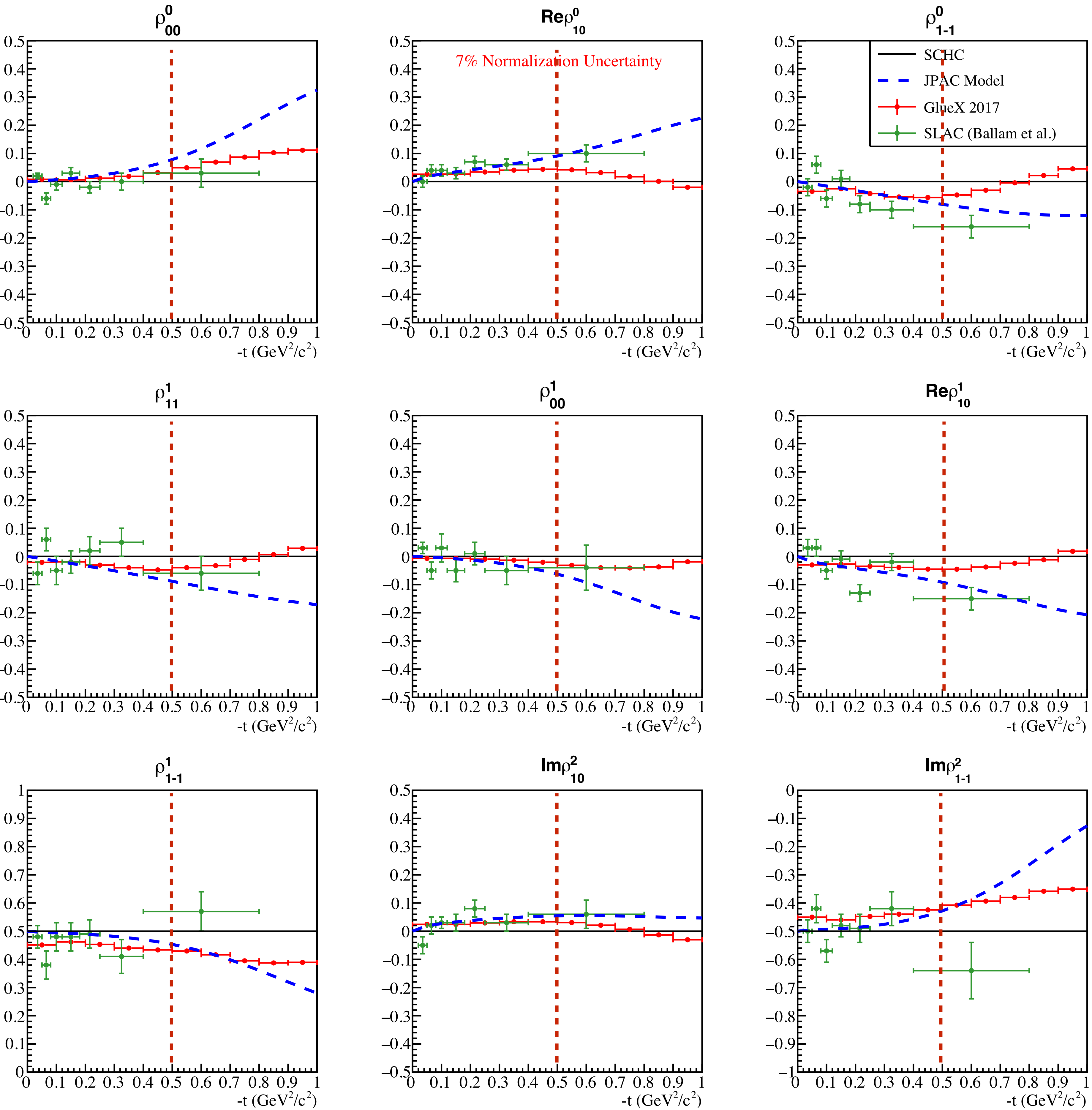
$$\rho^0_{1-1} \propto \beta_2\frac{-t}{m^2_\omega}$$

Predictions for Vector Meson SDME

VM et al (JPAC), PRD97 (2018)



$\vec{\gamma} p \to \rho^0 p$



Preliminary

Courtesy of A. Austregesilo

kinematics expanded

in power of $\dfrac{-t}{m_\rho^2}$

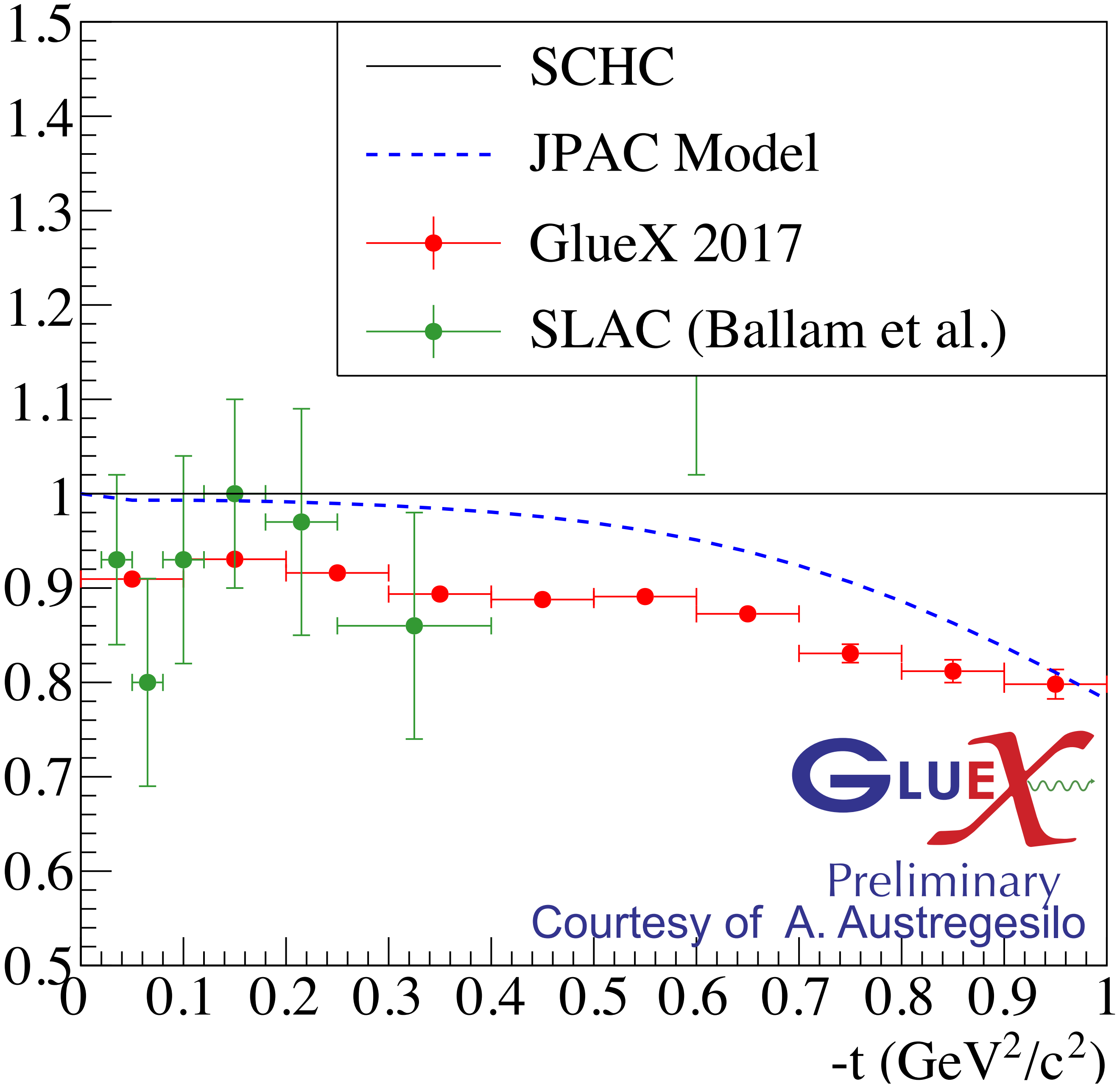
$$\rho^1_{1-1} = \pm\frac{1}{2} + \mathcal{O}(t^2)$$

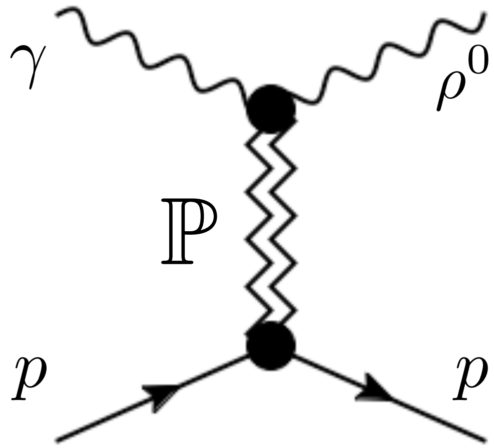
$$\text{Im}\ \rho^2_{1-1} = \mp\frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.

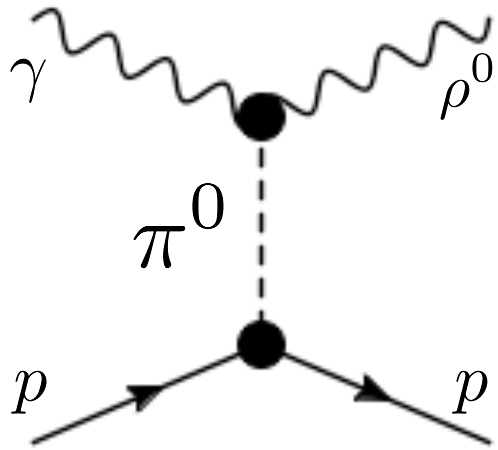
$\vec{\gamma} p \to \rho^0 p$

$$P_\sigma = 2\rho^1_{1-1} - \rho^1_{00}$$





$P_\sigma = +1$



$P_\sigma = -1$

small contribution

$P_\sigma \sim 0.9$ **is the ratio between natural vs unnatural exchanges**

Conclusion

Single Meson Photoproduction:

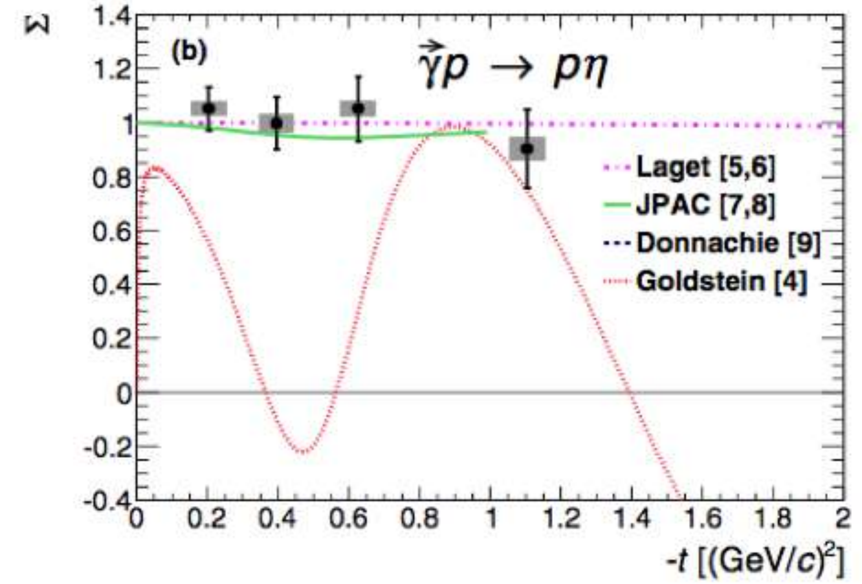
$$\vec{\gamma}p \to \pi^0 p$$

$$\vec{\gamma}p \to \eta p$$

$$\vec{\gamma}p \to \pi\Delta$$

Dominance of natural exch. in both π^0/η photoproduction

Significant $\pi^{\pm}$ exch. at low t



Vector Meson Photoproduction:

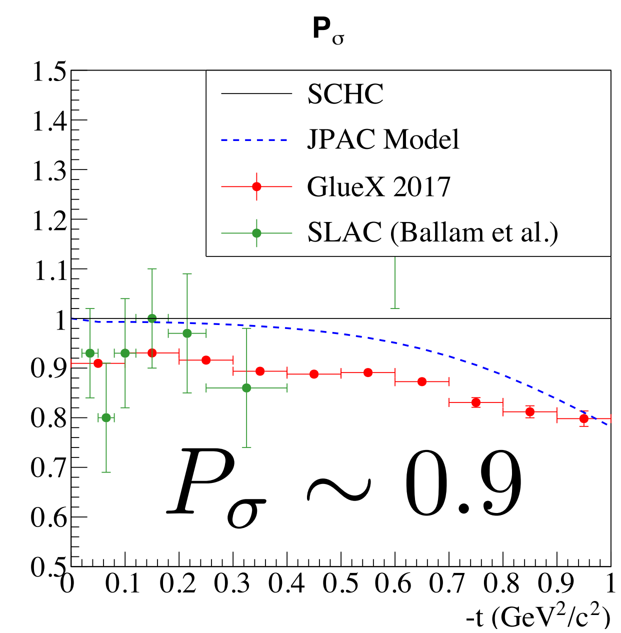
$$\vec{\gamma}p \to \rho^0 p$$

$$\vec{\gamma}p \to \omega p$$

$$\vec{\gamma}p \to \phi p$$

Consistent with factorization

Dominance of natural exchanges



Preliminary

Double Mesons Photoproduction:

$$\vec{\gamma}p \to \pi^0 \eta p$$

Observables: Moments of Angular distribution

$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega)\cos 2\Phi - P_\gamma I^2(\Omega)\sin 2\Phi$$

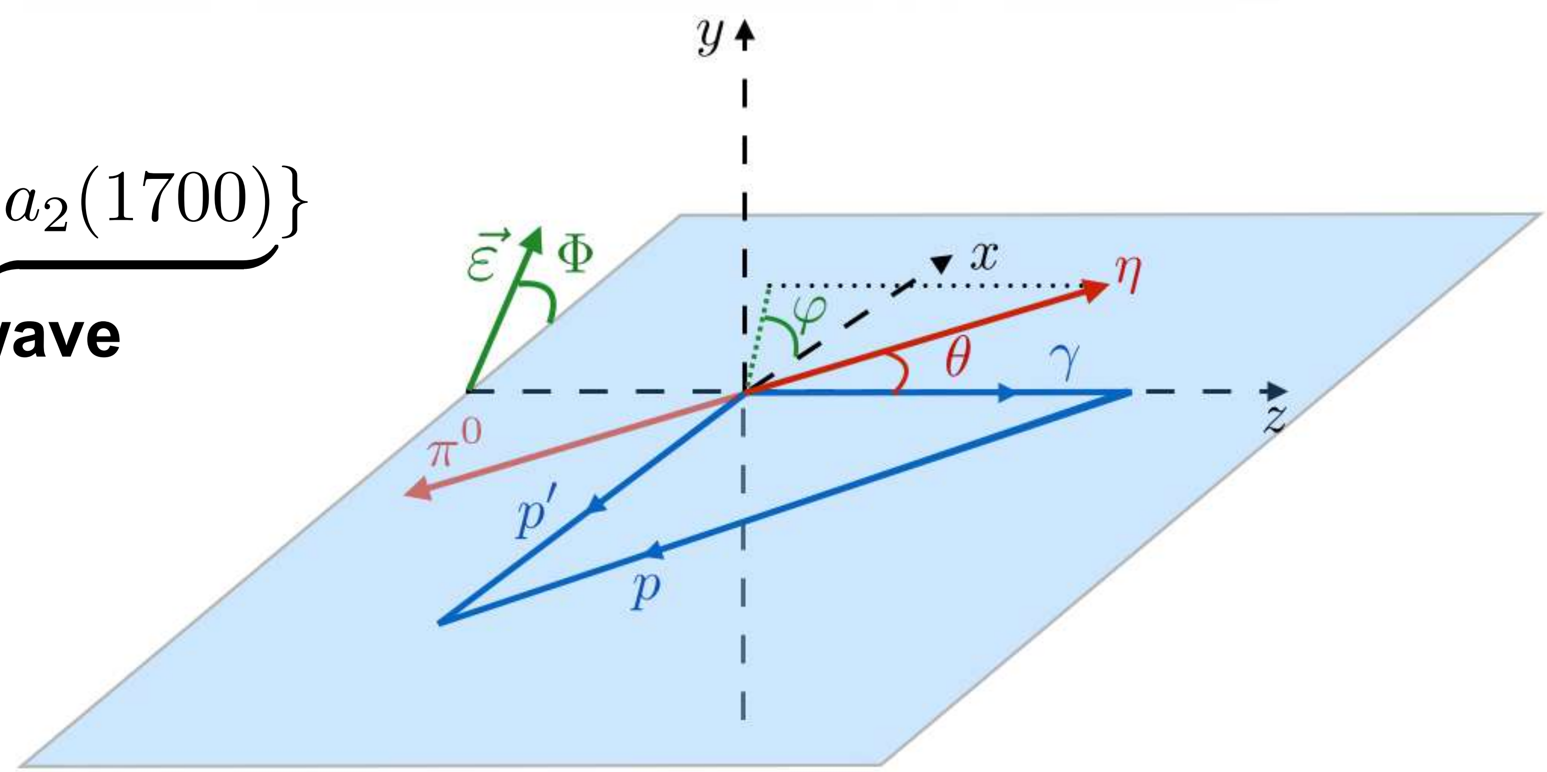
$$H^0(LM) = \frac{1}{2\pi}\int I(\Omega,\Phi)\, d^L_{M0}(\theta)\cos M\phi \,\mathrm{d}\Omega \mathrm{d}\Phi$$

$$R = \{\underbrace{a_0(980)}_{\text{S-wave}}, \underbrace{\pi_1(1600)}, \underbrace{a_2(1320), a_2(1700)}_{\text{D-wave}}\}$$

S-wave

D-wave

exotic P-wave



Observables: Moments of Angular distribution

$$H^0(LM) = \frac{1}{2\pi}\int I(\Omega,\Phi)\, d^L_{M0}(\theta)\cos M\phi\, \mathrm{d}\Omega\mathrm{d}\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma}\int I(\Omega,\Phi)\cos 2\Phi\, d^L_{M0}(\theta)\cos M\phi\, \mathrm{d}\Omega\mathrm{d}\Phi$$

$$\mathrm{Im}\, H^2(LM) = \frac{1}{\pi P_\gamma}\int I(\Omega,\Phi)\sin 2\Phi\, d^L_{M0}(\theta)\sin M\phi\, \mathrm{d}\Omega\mathrm{d}\Phi$$

Moments are unambiguously extracted and are related to partial waves (interferences)

$$^{(+)}H^0(00) = 2\left[|S_0^{(+)}|^2 + |P_{-1}^{(+)}|^2 + |P_0^{(+)}|^2 + |P_1^{(+)}|^2 + |D_{-2}^{(+)}|^2 + |D_{-1}^{(+)}|^2 + |D_0^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2\right]$$

$$^{(+)}H^0(10) = \frac{4}{\sqrt{3}}\mathrm{Re}\left(S_0^{(+)}P_0^{(+)*}\right) + \frac{8}{\sqrt{15}}\mathrm{Re}\left(P_0^{(+)}D_0^{(+)*}\right) + \frac{4}{\sqrt{5}}\mathrm{Re}\left(P_1^{(+)}D_1^{(+)*}\right) + \frac{4}{\sqrt{5}}\mathrm{Re}\left(P_{-1}^{(+)}D_{-1}^{(+)*}\right)$$

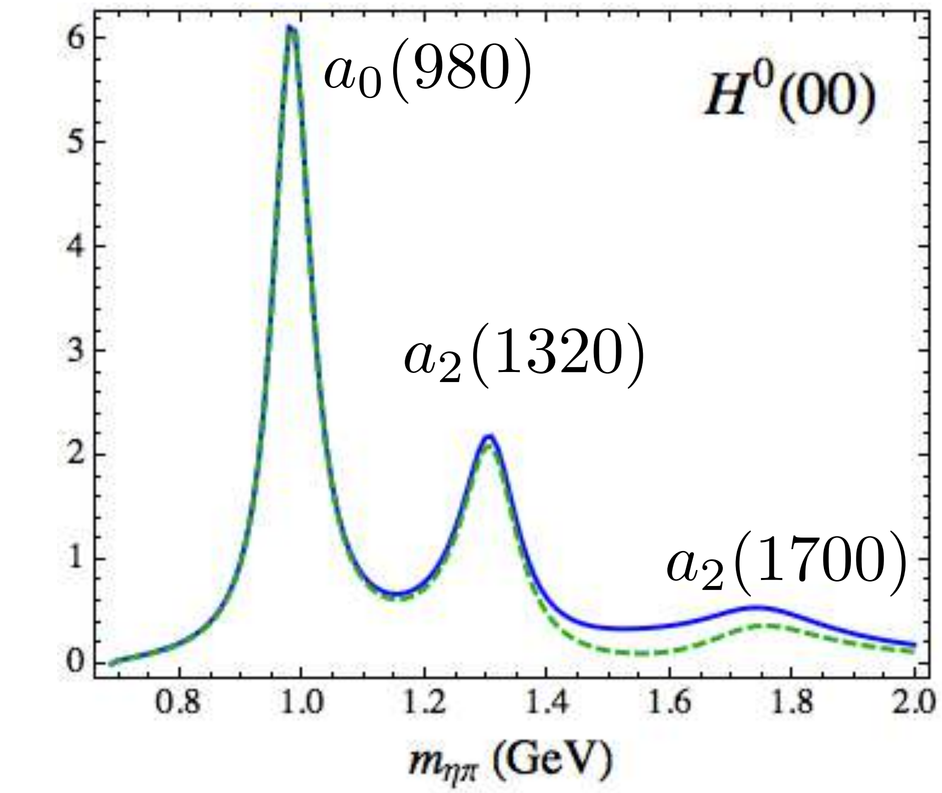
$$^{(+)}H^0(20) = \frac{4}{5}|P_0^{(+)}|^2 - \frac{2}{5}\left(|P_1^{(+)}|^2 + |P_{-1}^{(+)}|^2\right) + \frac{4}{7}|D_0^{(+)}|^2 + \frac{2}{7}\left(|D_1^{(+)}|^2 + |D_{-1}^{(+)}|^2\right)$$
$$- \frac{4}{7}\left(|D_2^{(+)}|^2 + |D_{-2}^{(+)}|^2\right) + \frac{4}{\sqrt{5}}\mathrm{Re}\left(S_0^{(+)}D_0^{(+)*}\right)$$

$$^{(+)}H^0(30) = \frac{12}{7\sqrt{5}}\mathrm{Re}\left(\sqrt{3}P_0^{(+)}D_0^{(+)*} - P_1^{(+)}D_1^{(+)*} - P_{-1}^{(+)}D_{-1}^{(+)*}\right)$$

Moments

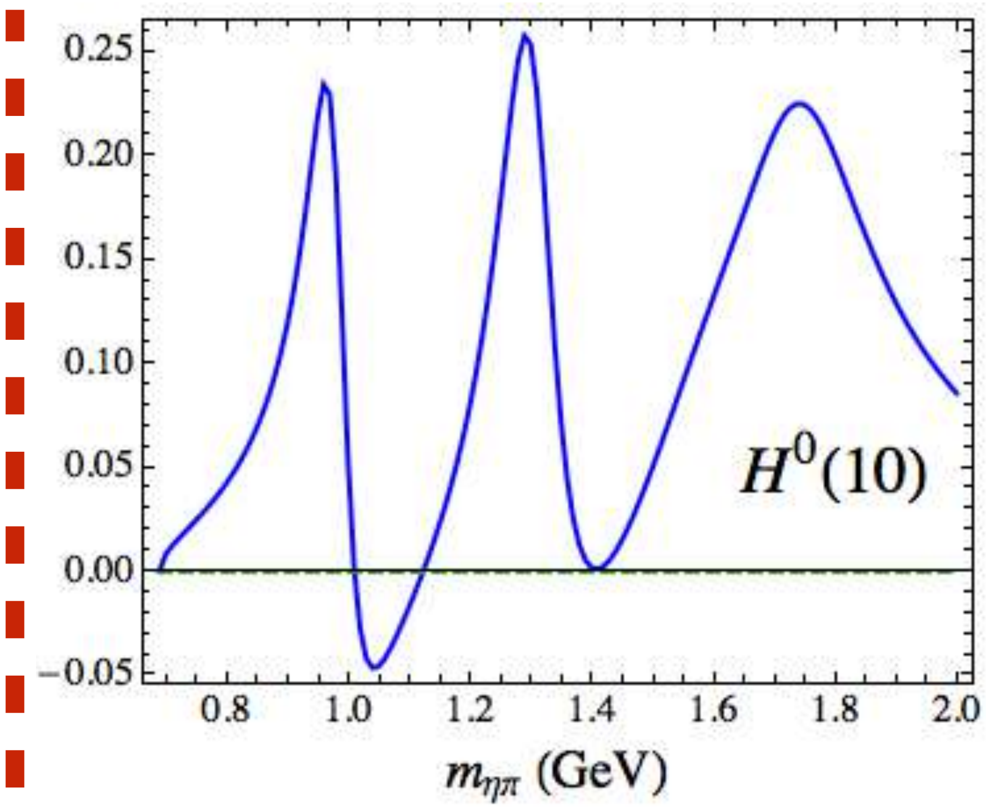
solid lines: *S* + *P* + *D* waves

dashed lines: *S* + *D* waves



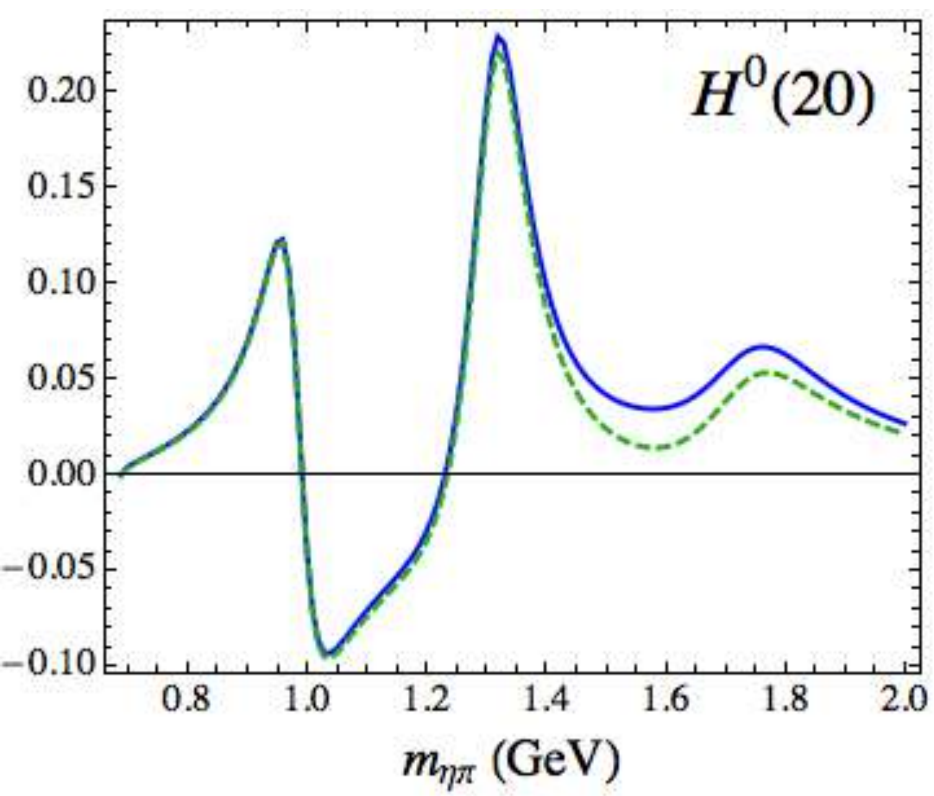
$$|S|^2 \quad |D|^2 \quad |P|^2$$

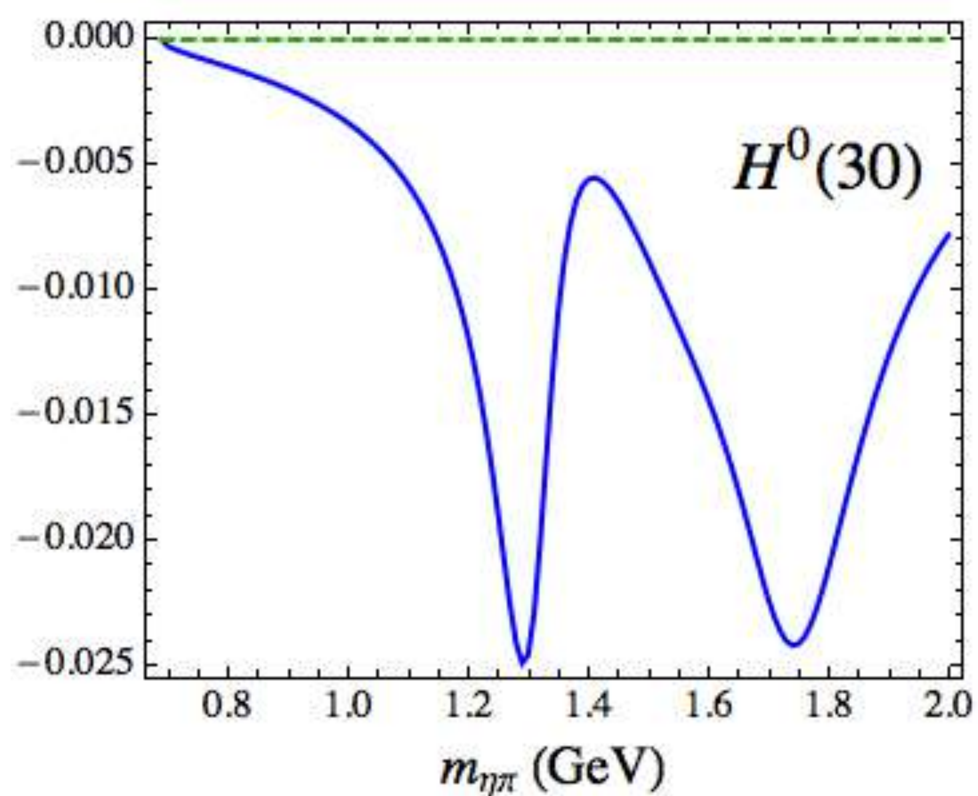
$$(S+D)(S+D)^*$$



$$(S+D)P^*$$

P- wave apparent as an interference in odd moments but not in even moments

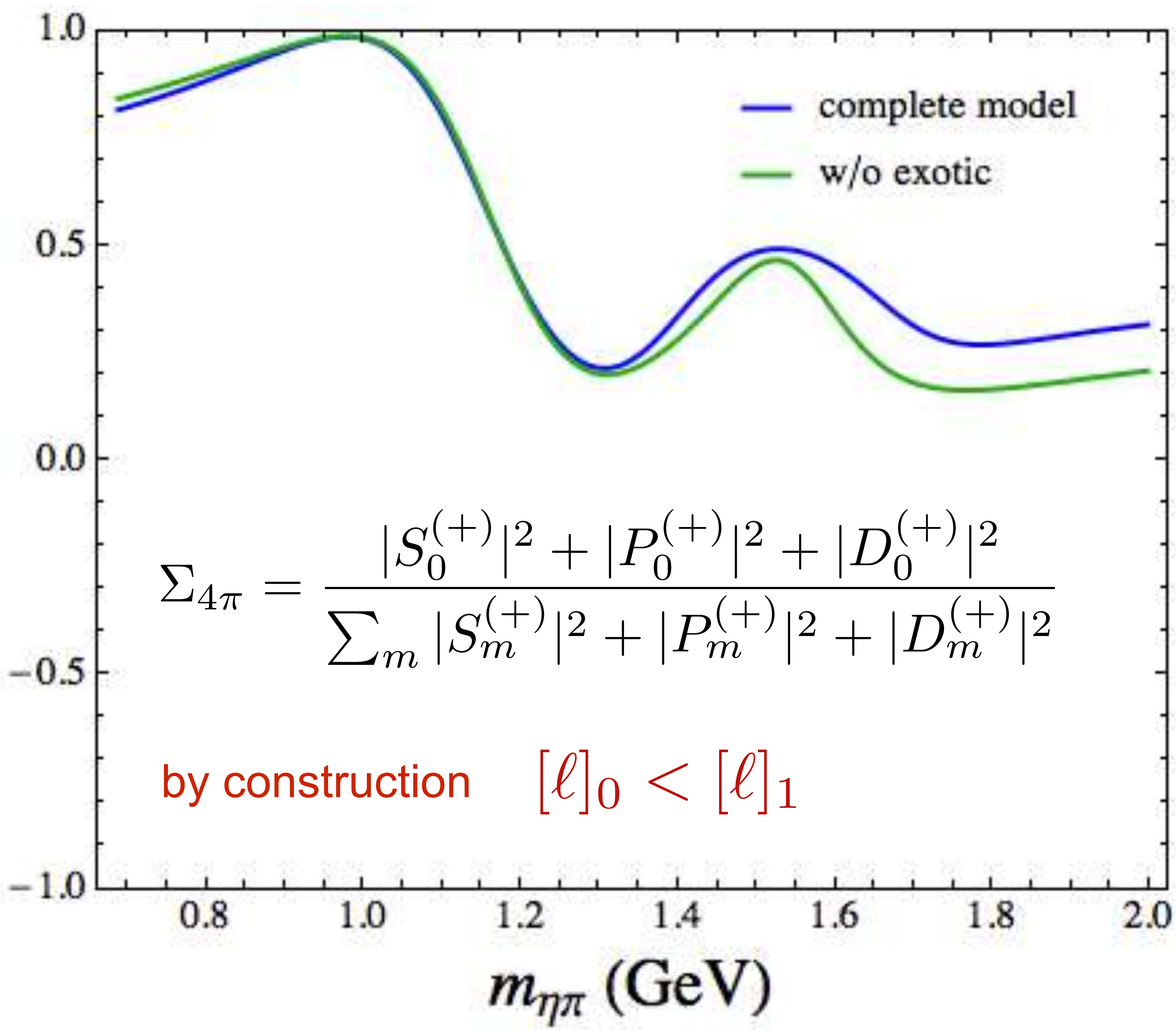




Beam Asymmetries

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) \mathrm{d}\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) \mathrm{d}\Omega}$$

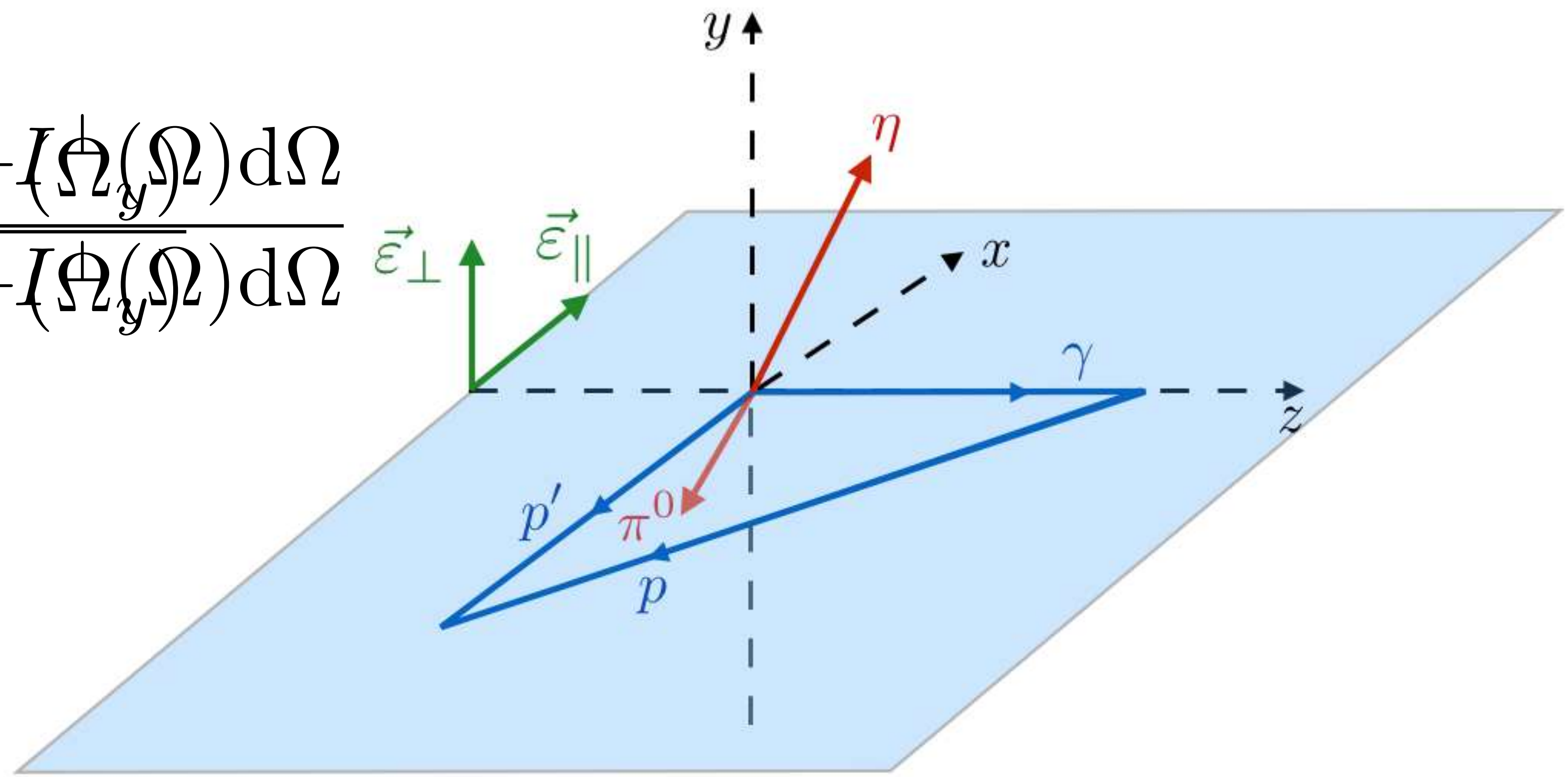
$\Sigma_{4\pi} =$ fully integrated



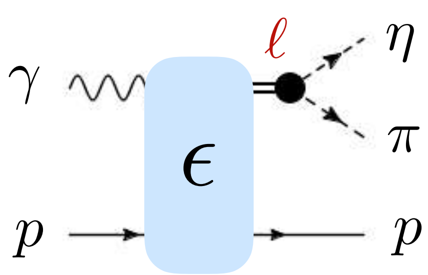
$$\Sigma_{4\pi} = \frac{|S_0^{(+)}|^2 + |P_0^{(+)}|^2 + |D_0^{(+)}|^2}{\sum_m |S_m^{(+)}|^2 + |P_m^{(+)}|^2 + |D_m^{(+)}|^2}$$

by construction $[\ell]_0 < [\ell]_1$

$$\Sigma_y = \frac{1}{P_\gamma}\frac{\int I_y^{\parallel}(\Omega) - I_y^{\perp}(\Omega)\,\mathrm{d}\Omega}{\int I_y^{\parallel}(\Omega) + I_y^{\perp}(\Omega)\,\mathrm{d}\Omega}$$



amplitude:
production x decay



Beam asymmetry sensitive to reflection through the reaction plane

use reflection operator = parity followed by 180° rotation around Y-axis

$$[\ell]_m^{(\epsilon)} \longrightarrow \Sigma_y = \epsilon(-1)^\ell$$

Odd waves change sign!!!

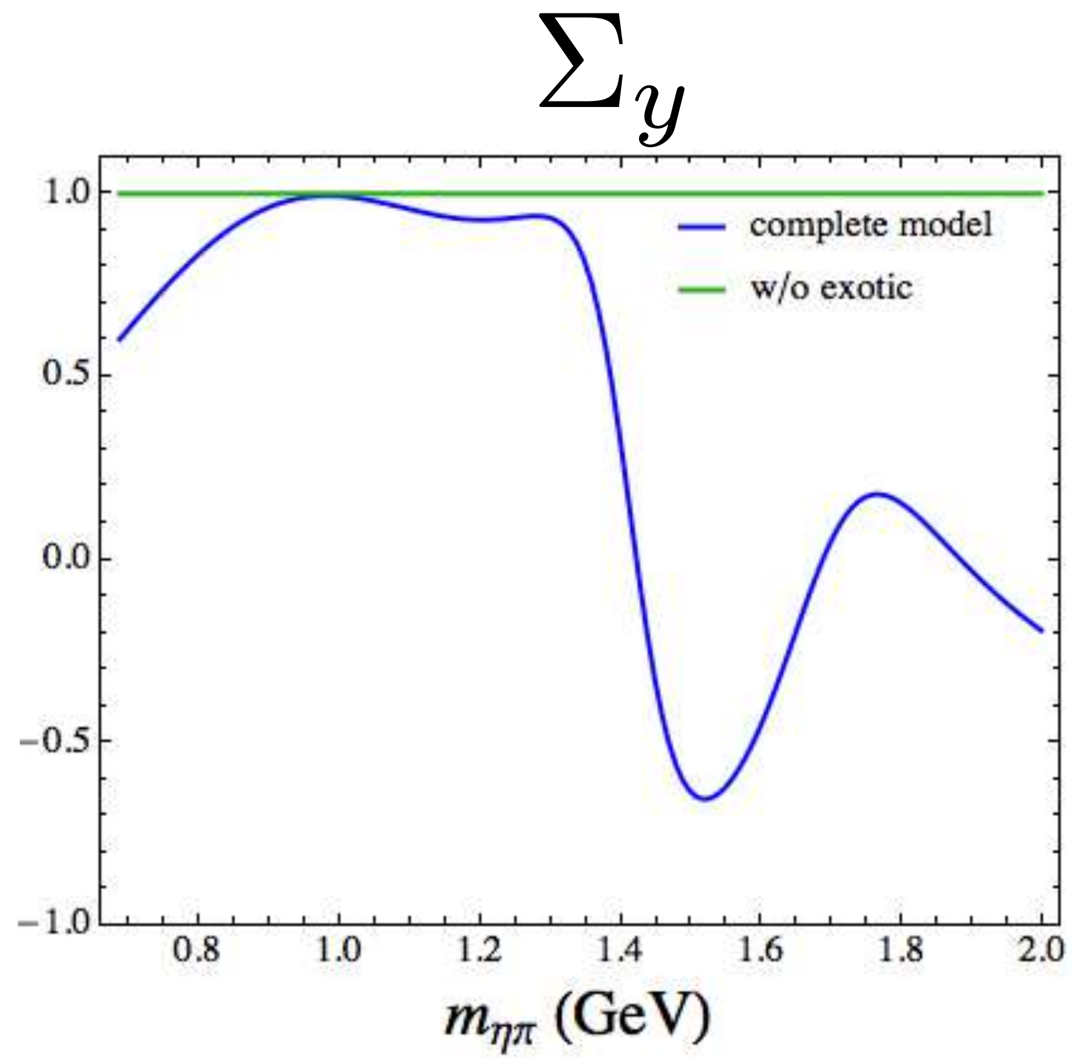
For $\pi^0\pi^0, \eta\eta$ **Only even waves** $\longrightarrow \Sigma_y = \epsilon$

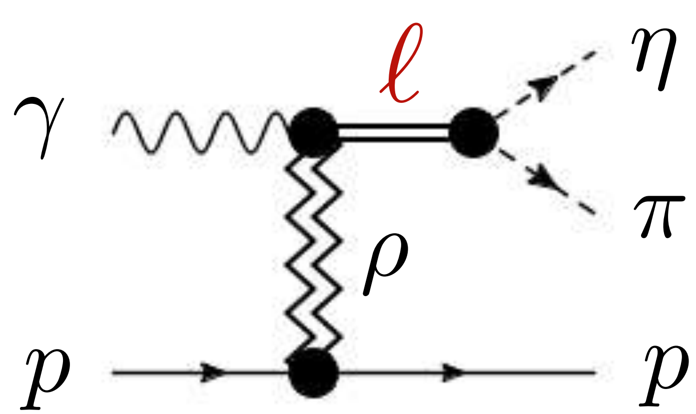
Beam Asymmetries

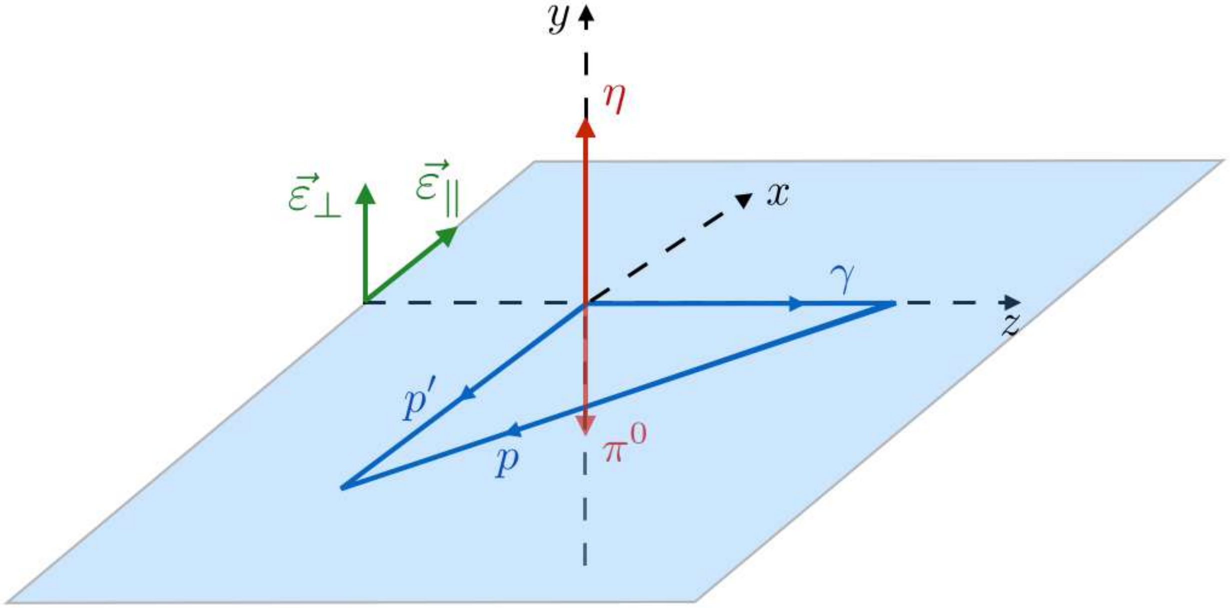
$$\Sigma_y = \frac{1}{P_\gamma}\frac{I(\Omega_y,0) - I(\Omega_y,\frac{\pi}{2})}{I(\Omega_y,0) + I(\Omega_y,\frac{\pi}{2})} = -\frac{I^1(\Omega_y)}{I^0(\Omega_y)}$$

Intensities can be computed from moments:

$$I^0(\Omega_y) = H^0(00) - \frac{5}{2}H^0(20) - 5\sqrt{\frac{3}{2}}H^0(22) + \frac{27}{8}H^0(40) + \frac{9}{2}\sqrt{\frac{5}{2}}H^0(42) + \frac{9}{4}\sqrt{\frac{35}{2}}H^0(44)$$





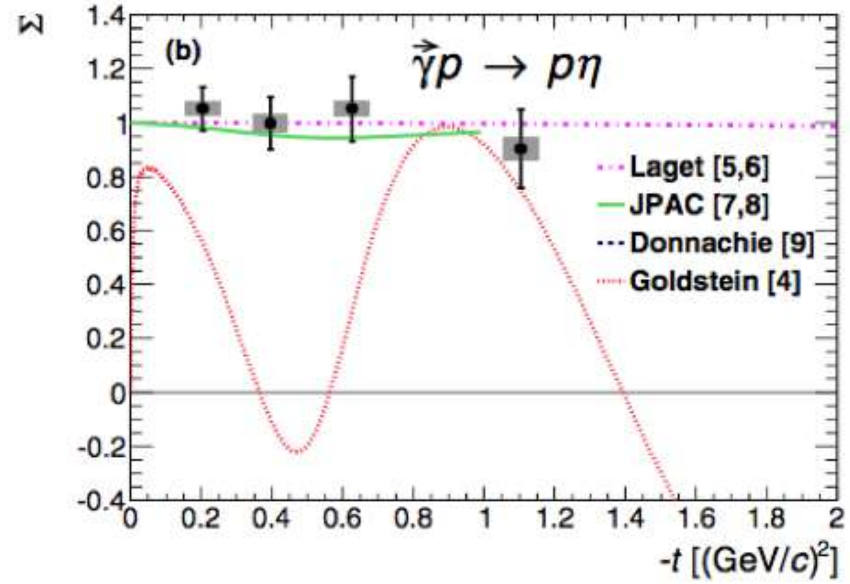


Conclusion

Single Meson Photoproduction:

$$\vec{\gamma}p \to \pi^0 p$$
$$\vec{\gamma}p \to \eta p$$

Dominance of natural exchanges in both π^0/η photoproduction



Vector Meson Photoproduction:

$$\vec{\gamma}p \to \rho^0 p$$
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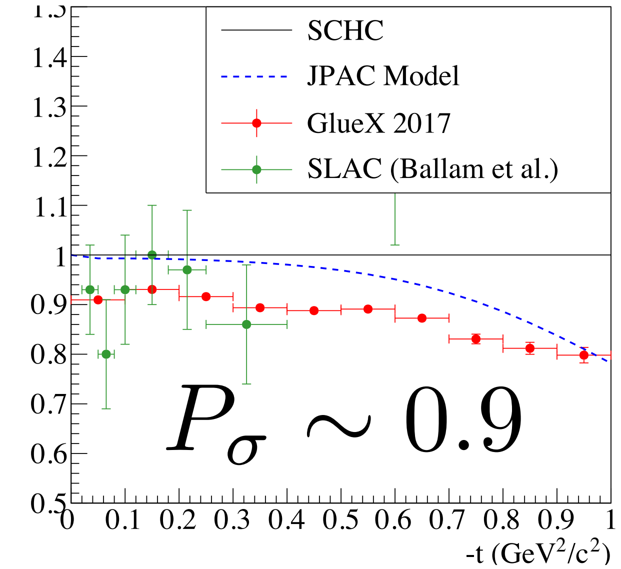
Consistent with factorization

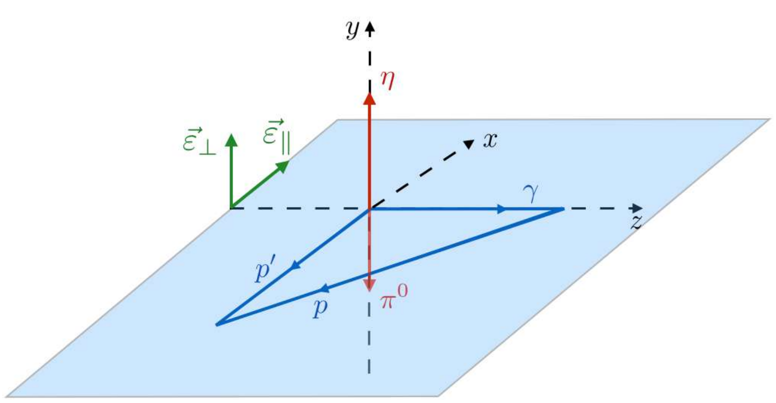
Dominance of natural exchanges

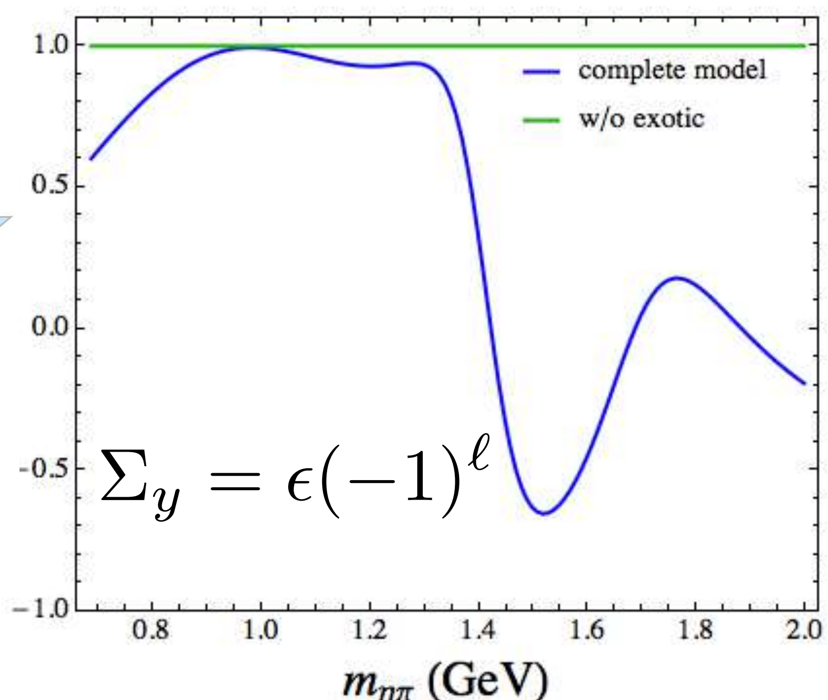


Double Mesons Photoproduction:

$$\vec{\gamma}p \to \pi^0 \eta p$$

New observable sensitive to exotic production

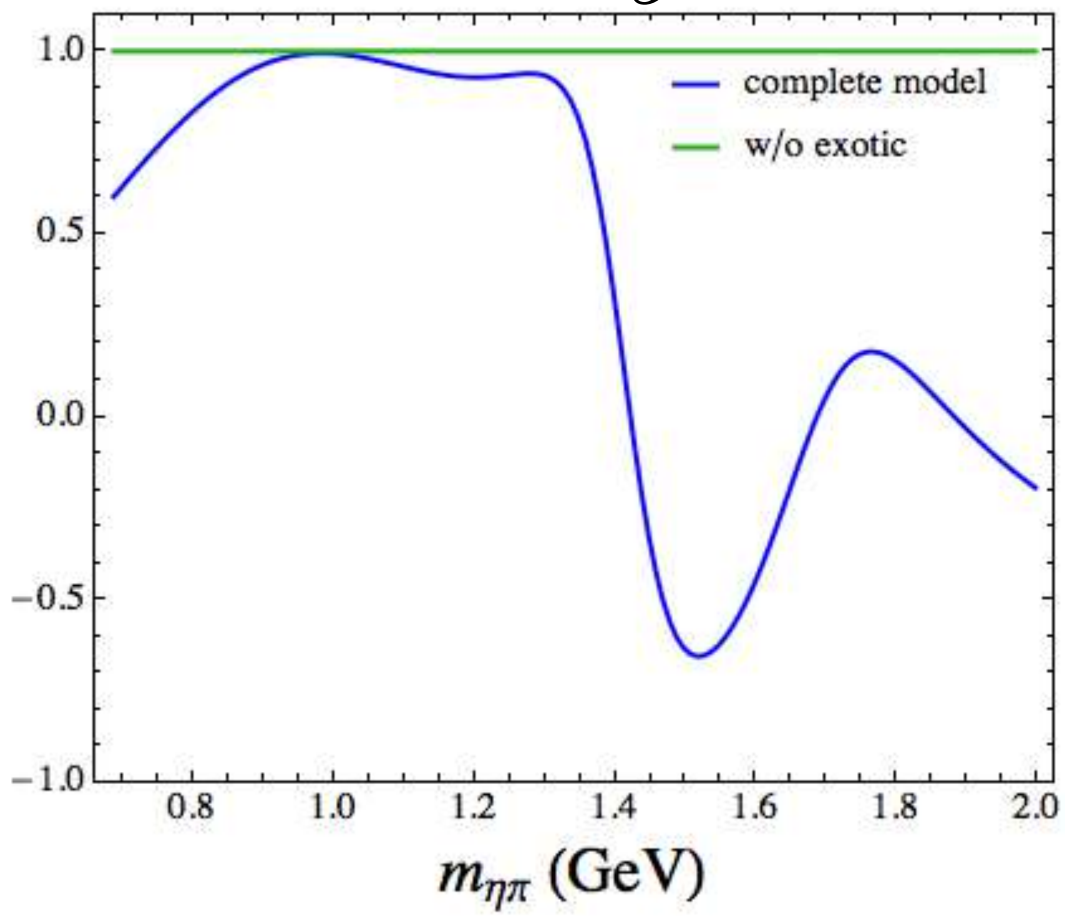


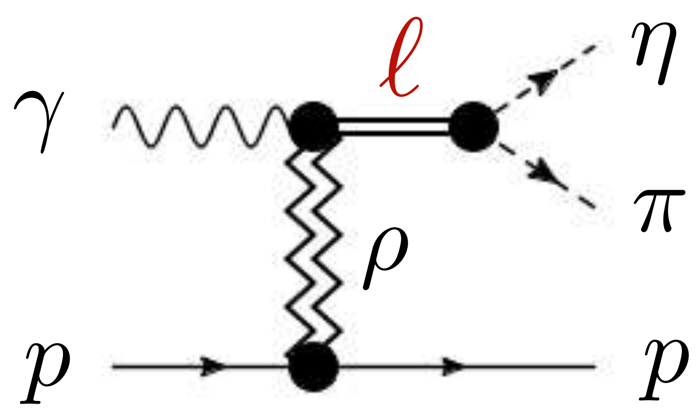


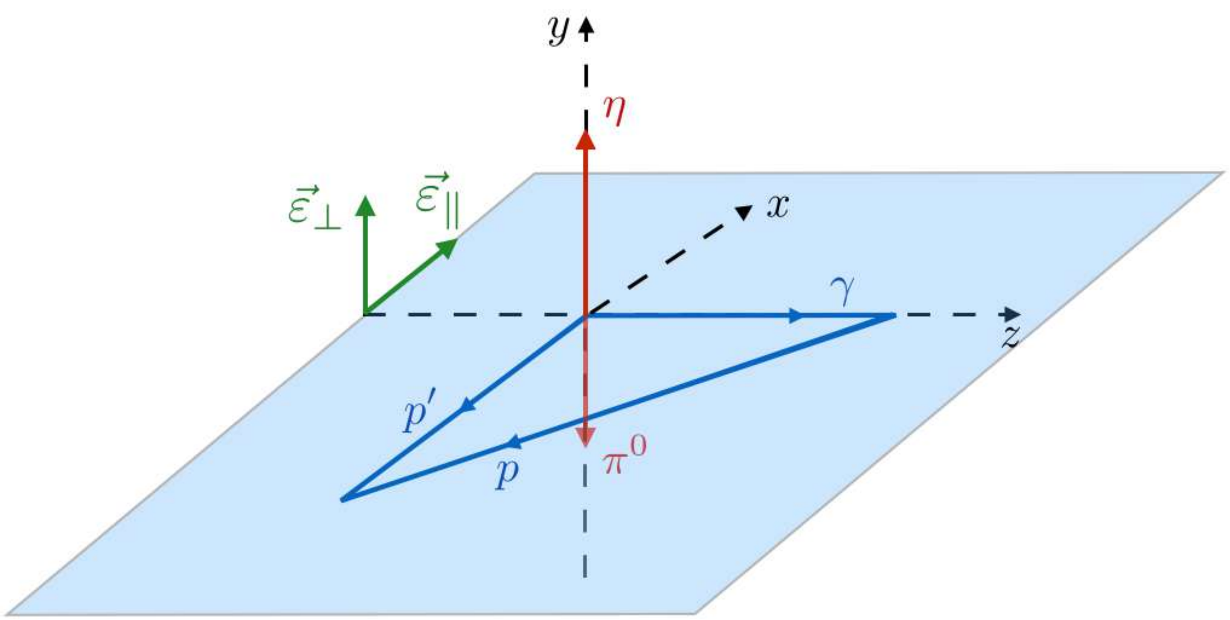
Backup Slides

Beam Asymmetries

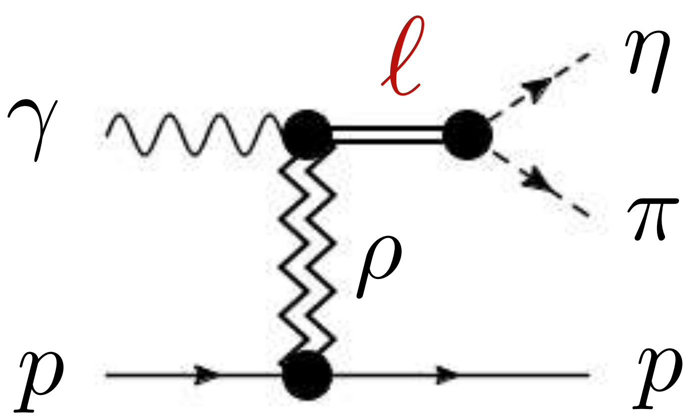
Σ_y







Model



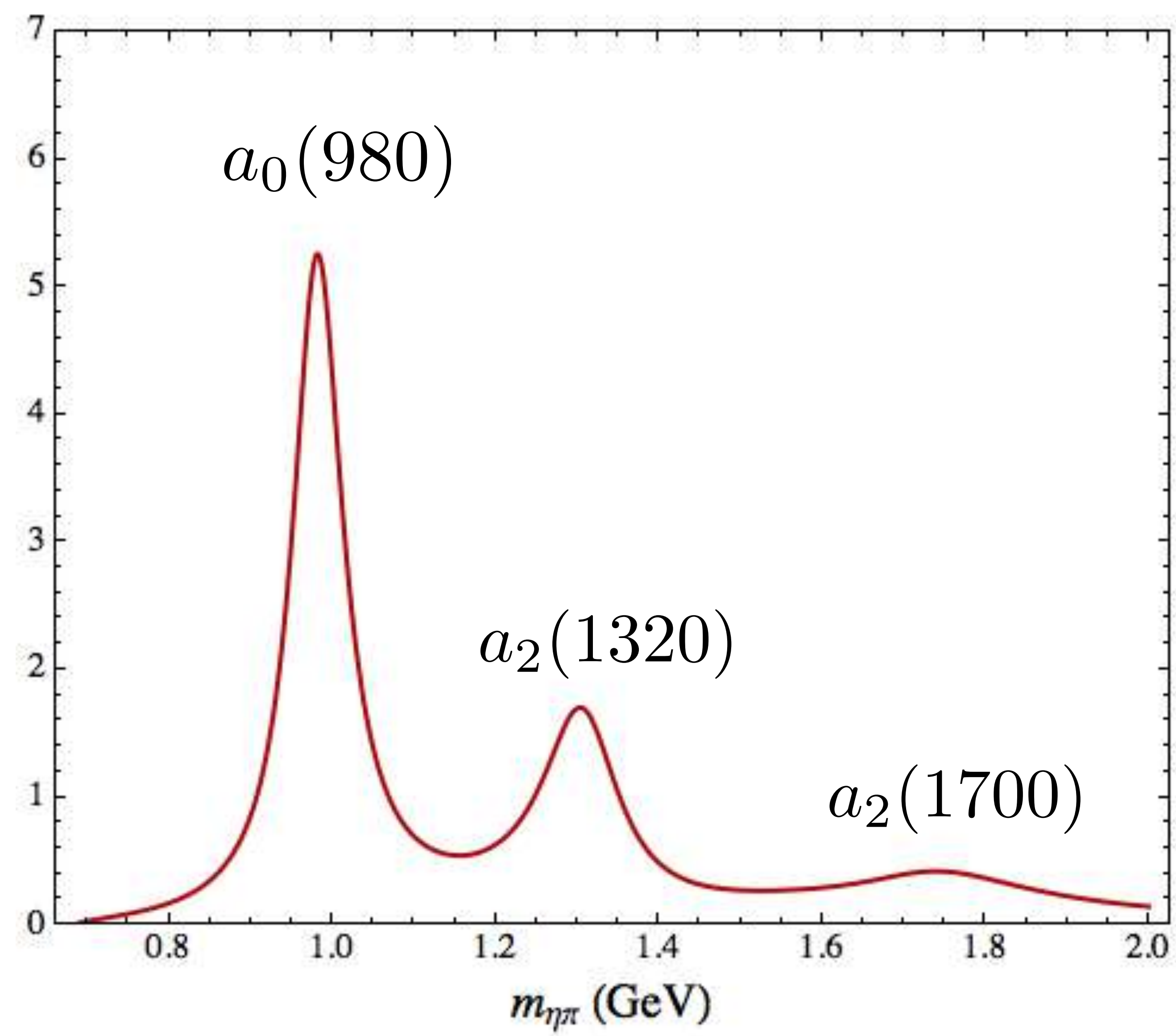
$$R = \{\underbrace{a_0(980)}_{S_0^{(+)}}, \underbrace{\pi_1(1600)}_{P_{0,1}^{(+)}}, \underbrace{a_2(1320), a_2(1700)}_{D_{0,1,2}^{(+)}}\}$$

production: natural exchanges

line shape: Breit-Wigner form

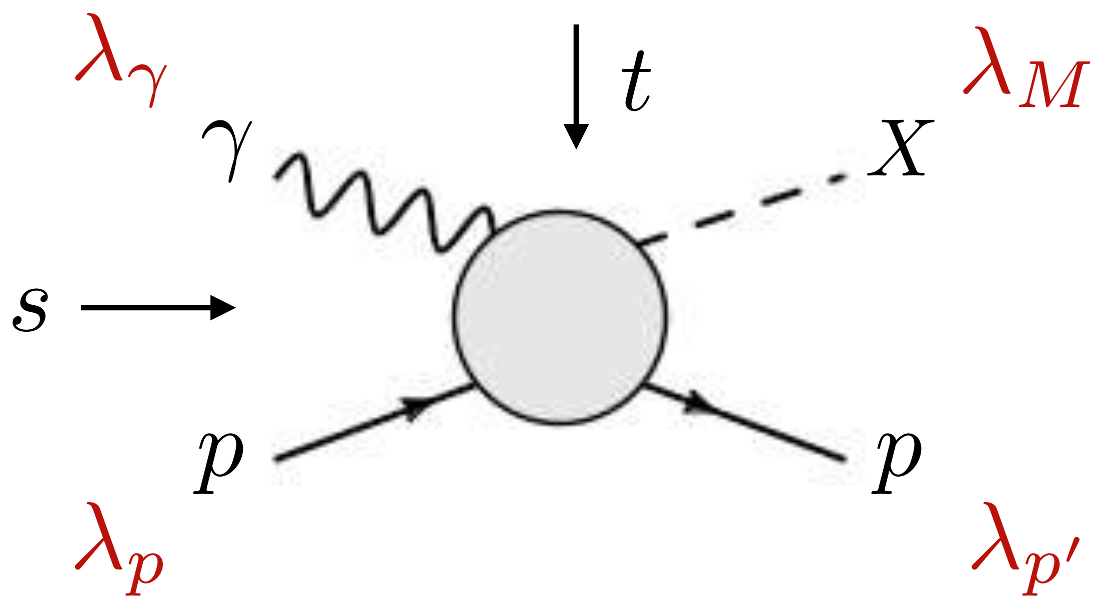
parameters: arbitrary

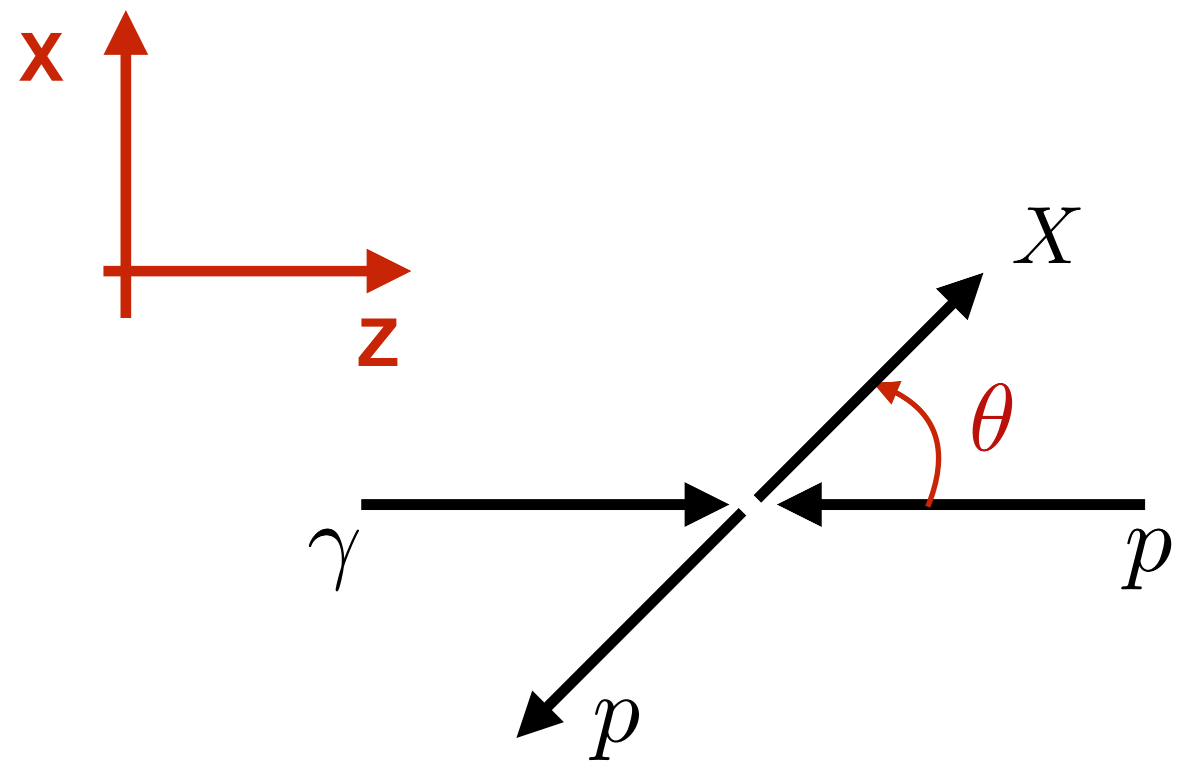
Small exotic wave,
not apparent in the diff. cross. section



VM et al (JPAC), in preparation

Kinematics





$$A^{\lambda_\gamma \lambda_M}_{\lambda_p \lambda_{p'}}(s,t)$$

$\lambda_i =$ **s-channel helicity of particle i**

$t =$ **momentum transfered squared**

$s =$ **center of mass energy squared**

High energy approximation

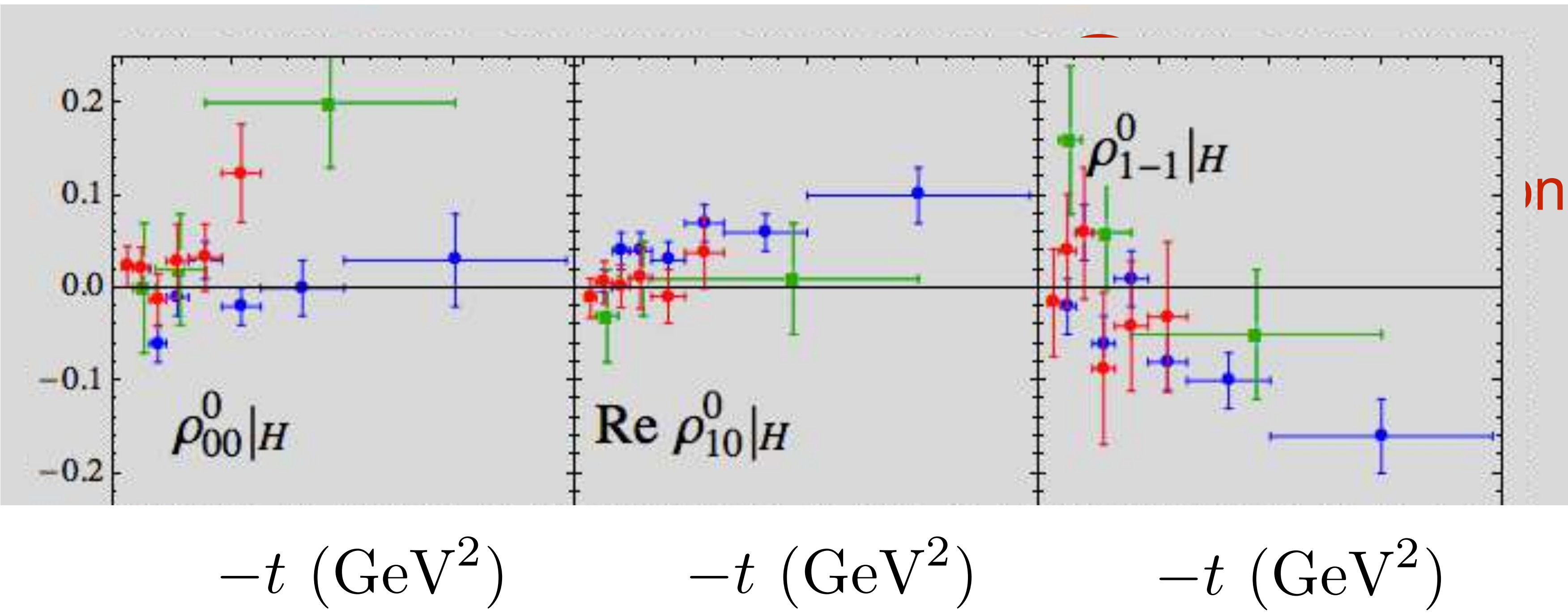
$$\cos\theta \to 1 + \frac{2t}{s}$$

$$\sin\theta \to 2\sqrt{-t/s}$$

$$\sin\theta/2 \to \sqrt{-t/s}$$

Spin Density Matrix Elements at 9.3 GeV (SLAC)

$\vec{\gamma}p \to \rho p$ $\qquad\qquad$ $\vec{\gamma}p \to \omega p$



n of factorization?

$$\rho^0_{00} \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

$$\mathrm{Re}\ \rho^0_{10} \propto \frac{1}{2}\beta_1 \frac{\sqrt{-t}}{m_\omega}$$

$$\rho^0_{1-1} \propto \beta_2 \frac{-t}{m_\omega^2}$$

Rho data seems consistent

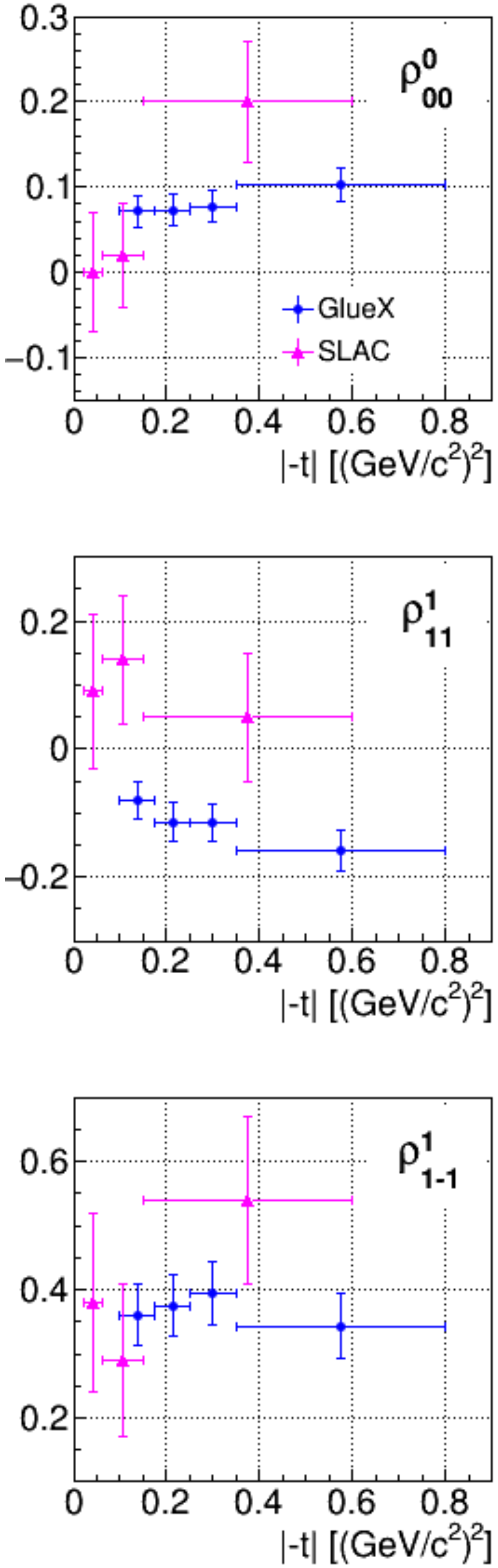
Omega data more problematic

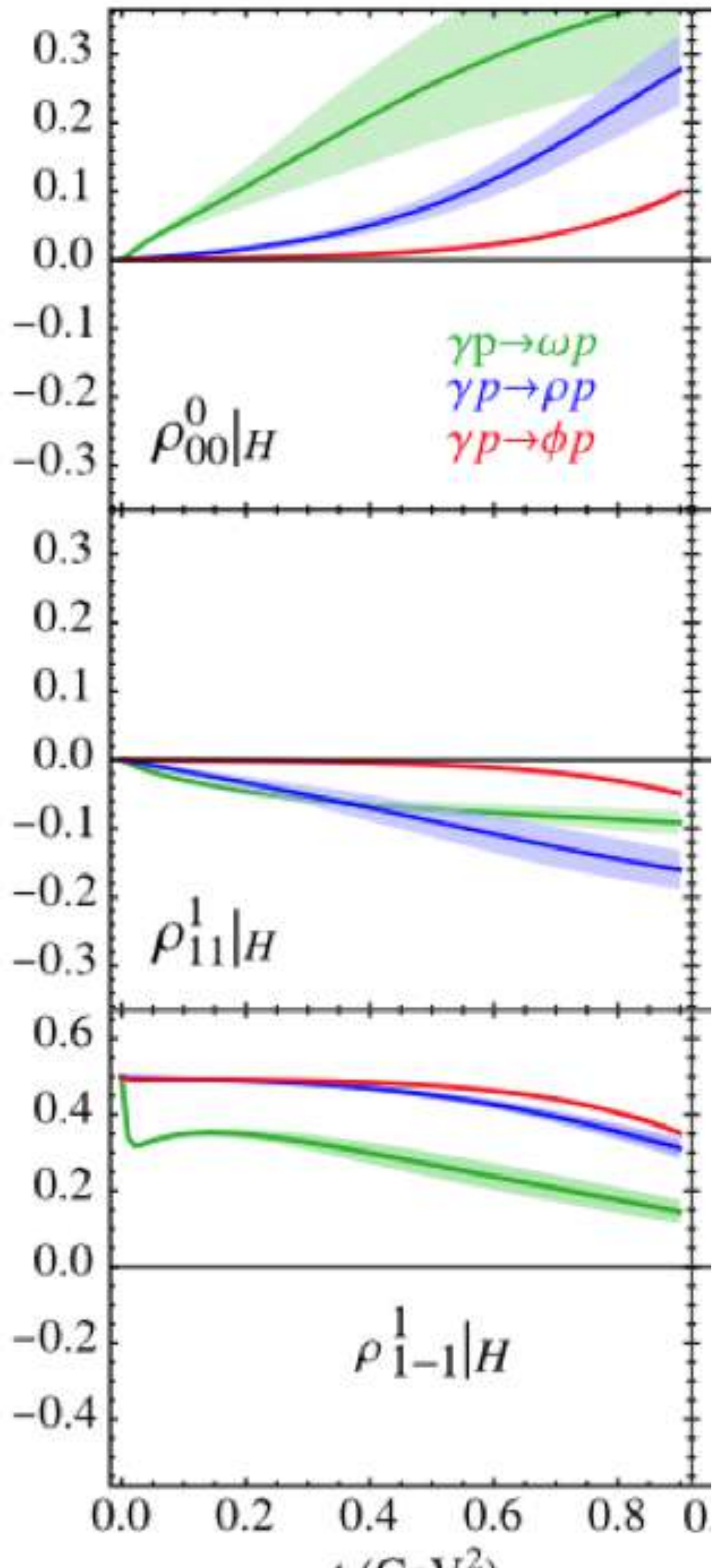
$$\rho^1_{1-1} = \pm\frac{1}{2} + \mathcal{O}(t^2)$$

$$\mathrm{Im}\ \rho^2_{1-1} = \mp\frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.

$\vec{\gamma} p \to \omega p$





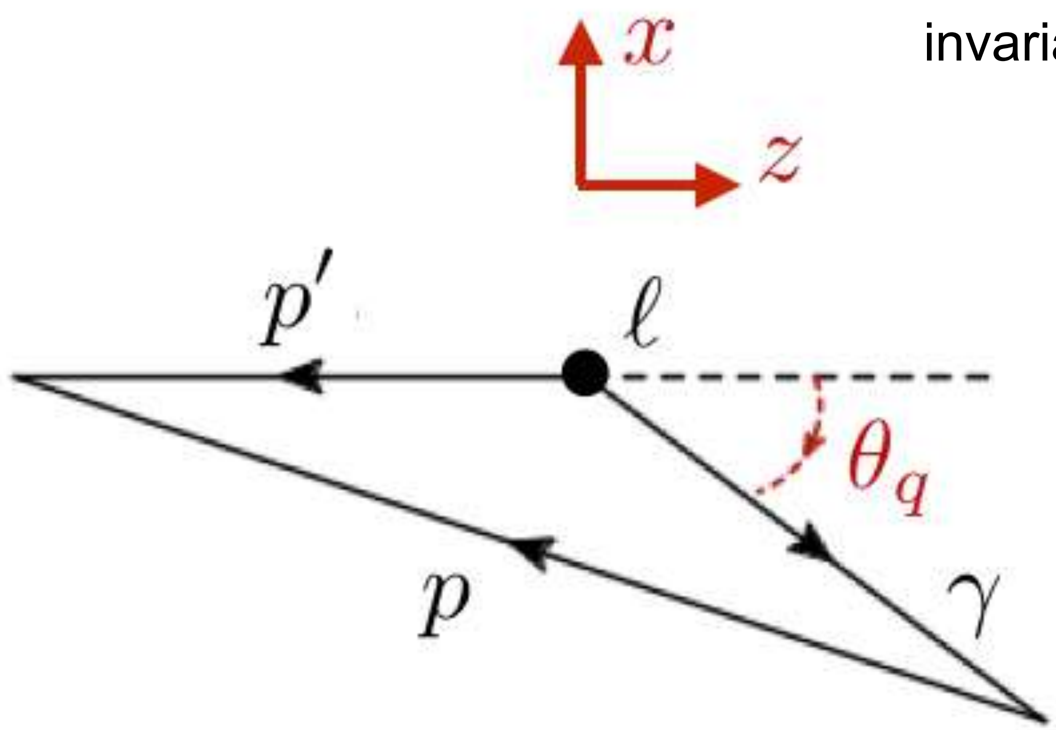
purple points: SLAC

blue points: GlueX

Preliminary

Frame

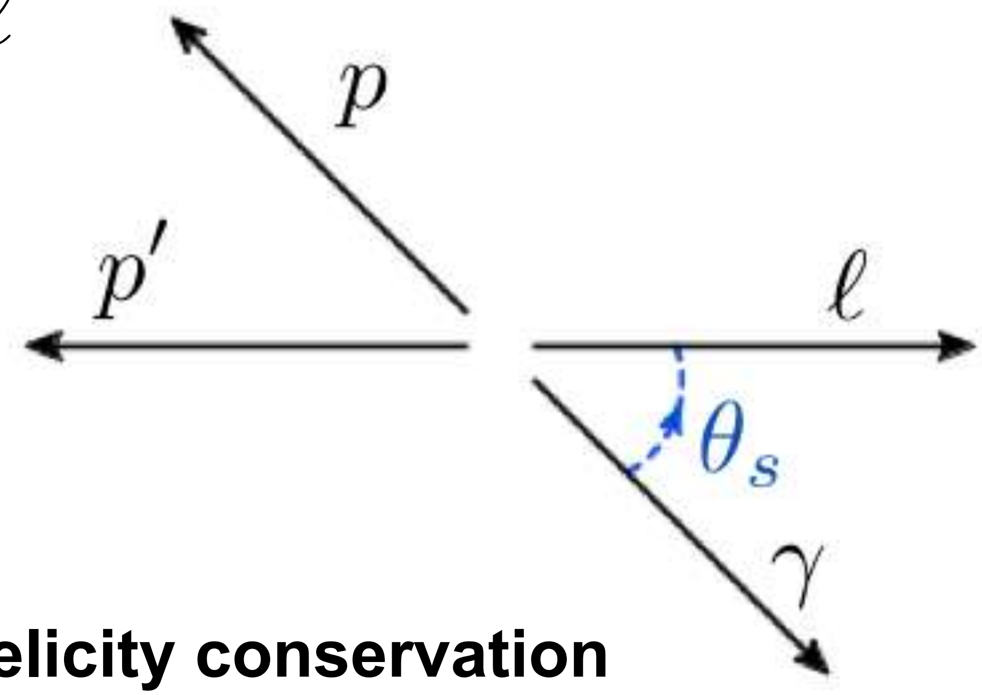
Helicity frame



boost

invariant helicity ℓ

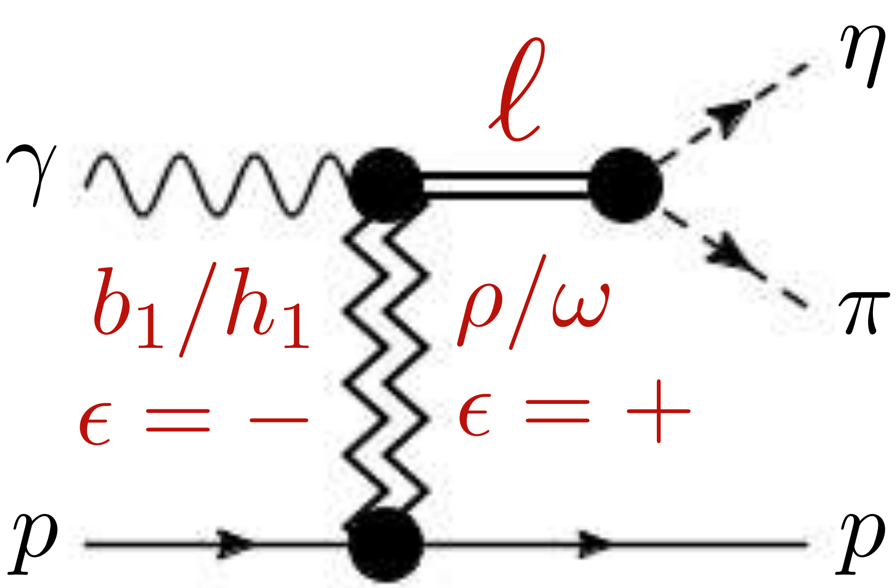
s-channel



helicity conservation

between γ **and** ℓ

$$T_{\lambda_\gamma m} \simeq \delta_{\lambda_\gamma, m} T_{\lambda_\gamma m} + \dots$$



Reflectivity basis:

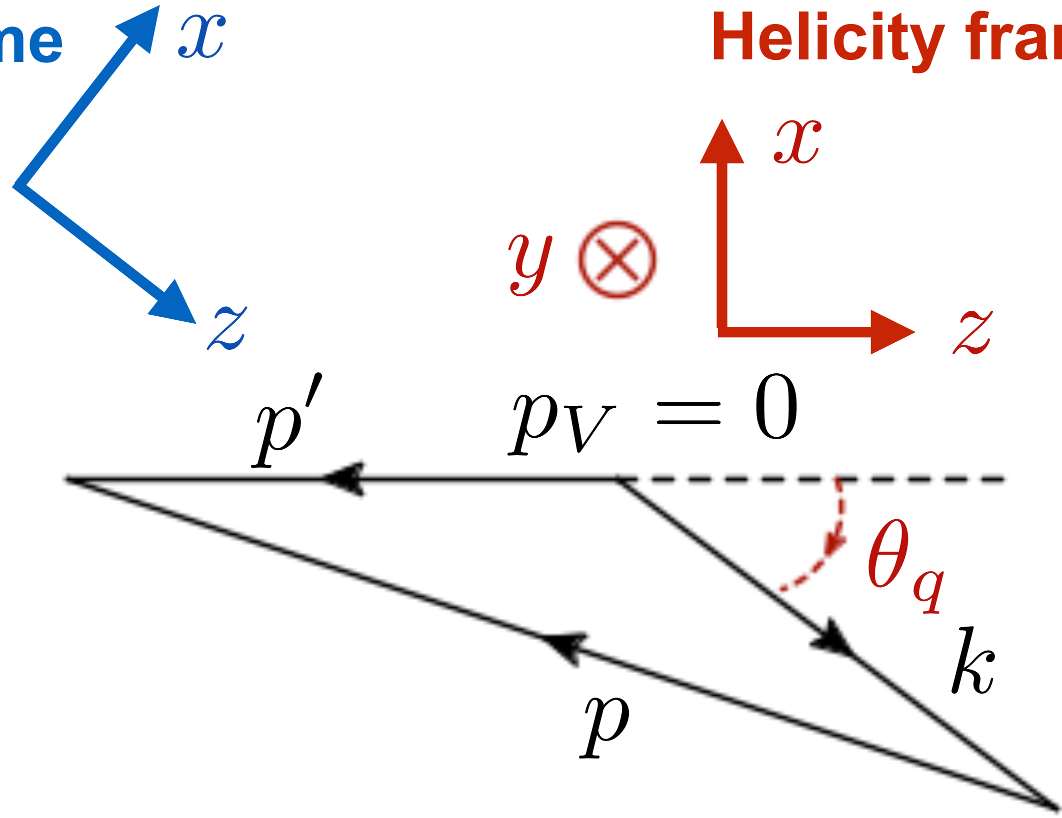
$$[\ell]_m^{(\epsilon)} = T_{1m} - \epsilon\, T_{-1-m}$$

Dominant: $(\epsilon = +, m = 1)$

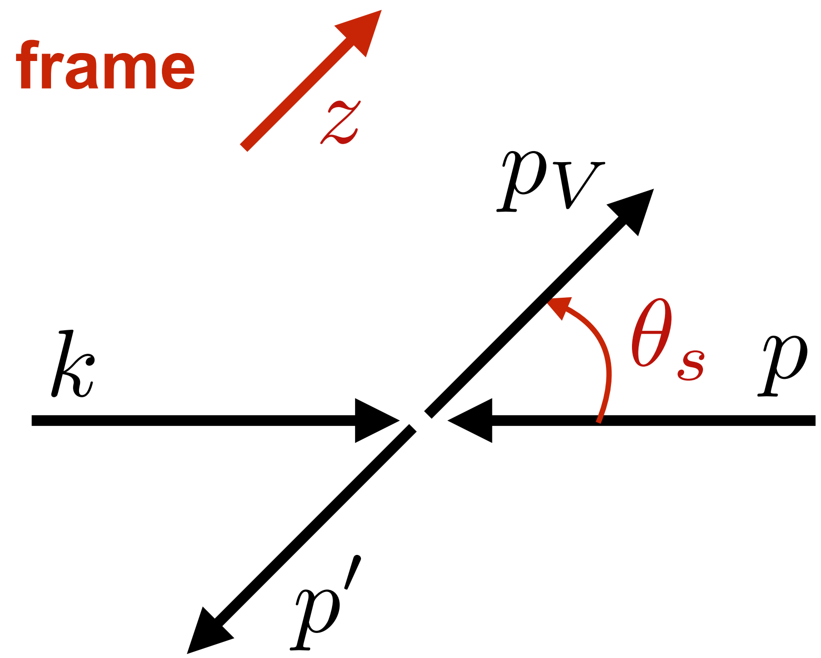
Frames

GJ frame

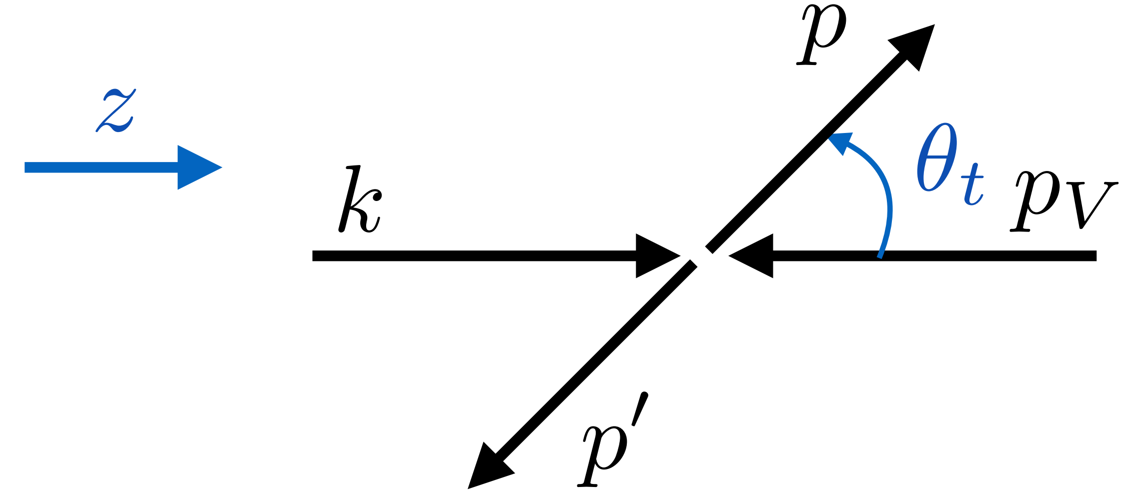
Helicity frame



s-channel frame



t-channel frame



$$\rho_{MM'}\big|_H = \rho_{MM'}\big|_{s\text{-chan}}$$

rotation

$$\rho_{MM'}\big|_{GJ} = \rho_{MM'}\big|_{t\text{-chan}}$$

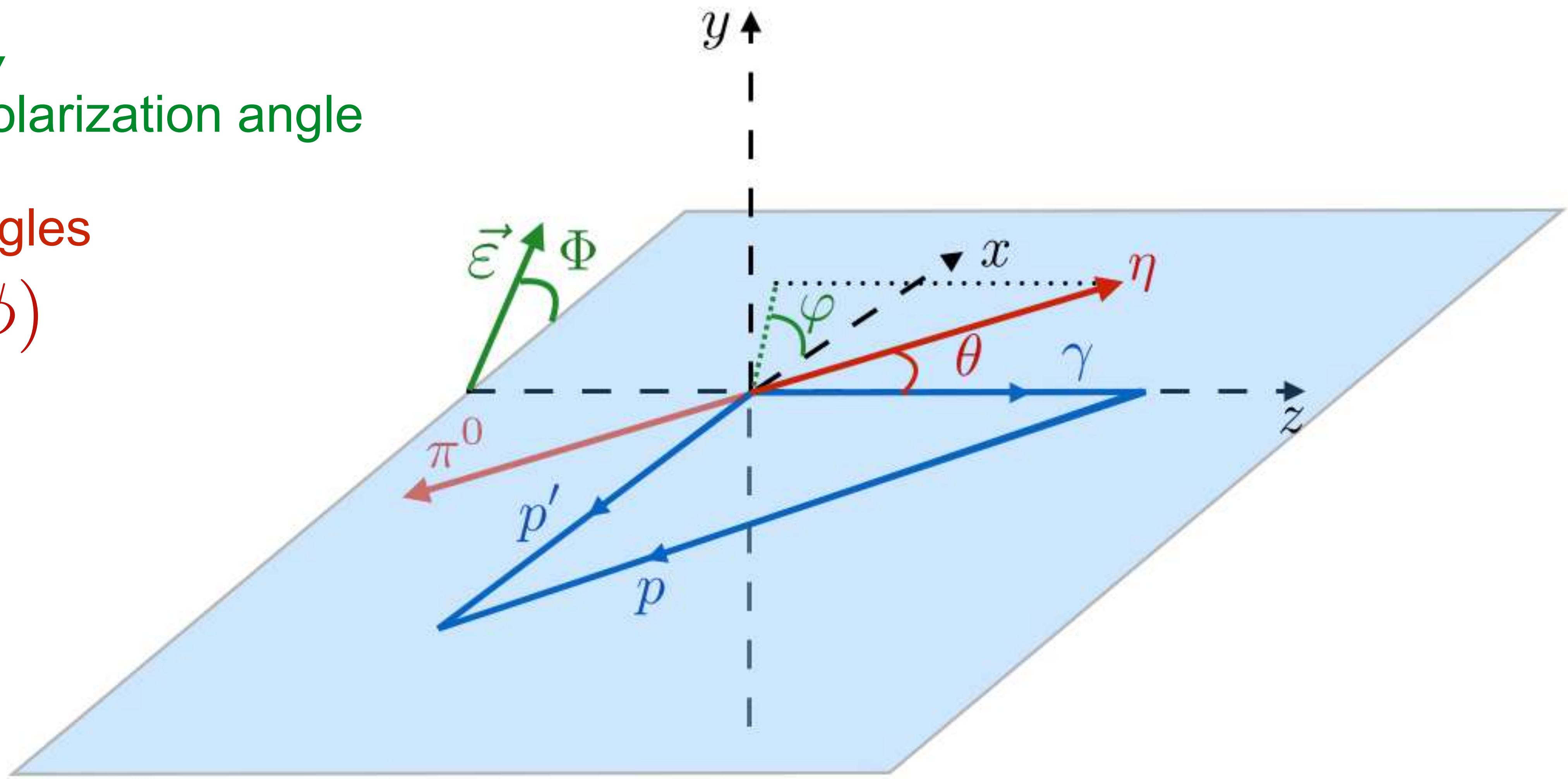
Measured Intensities

$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega)\cos 2\Phi - P_\gamma I^2(\Omega)\sin 2\Phi$$

polarization angle

η decay angles

$\Omega = (\theta, \phi)$

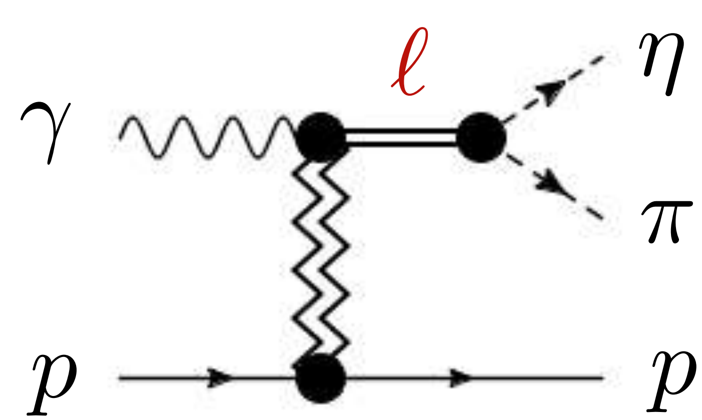


Implicit variables

Beam energy (fixed)

momentum transfer (integrated)

$\eta\pi$ invariant mass (binned)



Spin Density Matrix Elements

$$\rho^0_{00} = \frac{2}{N} \sum_{\lambda,\lambda'} \left| T_{\substack{1,0 \\ \lambda,\lambda'}} \right|^2$$

$$N = \sum_{\lambda,\lambda',\lambda_\gamma,\lambda_V} \left| T_{\substack{\lambda_\gamma,\lambda_V \\ \lambda,\lambda'}} \right|^2$$

$$\mathrm{Re}\ \rho^0_{10} = \frac{1}{N} \mathrm{Re} \sum_{\lambda,\lambda'} \left(T_{\substack{1,1 \\ \lambda,\lambda'}} - T_{\substack{1,-1 \\ \lambda,\lambda'}} \right) T^*_{\substack{1,0 \\ \lambda,\lambda'}}$$

$$\rho^1_{11} = \frac{2}{N} \mathrm{Re} \sum_{\lambda,\lambda'} T_{\substack{-1,1 \\ \lambda,\lambda'}} T^*_{\substack{1,1 \\ \lambda,\lambda'}}$$

$$\rho^0_{1-1} = \frac{2}{N} \mathrm{Re} \sum_{\lambda\lambda'} T_{\substack{1,1 \\ \lambda,\lambda'}} T^*_{\substack{1,-1 \\ \lambda,\lambda'}}$$

$$\rho^1_{00} = \frac{2}{N} \mathrm{Re} \sum_{\lambda,\lambda'} T_{\substack{-1,0 \\ \lambda,\lambda'}} T^*_{\substack{1,0 \\ \lambda,\lambda'}}$$

$$\rho^1_{1-1} + \mathrm{Im}\ \rho^2_{1-1} = \frac{2}{N} \sum_{\lambda,\lambda'} T_{\substack{-1,1 \\ \lambda,\lambda'}} T^*_{\substack{1,-1 \\ \lambda,\lambda'}}$$

$$\mathrm{Re}\ \rho^1_{10} + \mathrm{Im}\ \rho^2_{10} = \frac{1}{N} \mathrm{Re} \sum_{\lambda,\lambda'} \mathcal{M}_{\substack{-1,1 \\ \lambda,\lambda'}} \mathcal{M}^*_{\substack{1,0 \\ \lambda,\lambda'}}$$

$$\rho^1_{1-1} - \mathrm{Im}\ \rho^2_{1-1} = \frac{2}{N} \sum_{\lambda,\lambda'} T_{\substack{1,1 \\ \lambda,\lambda'}} T^*_{\substack{-1,-1 \\ \lambda,\lambda'}}$$

$$\mathrm{Re}\ \rho^1_{10} - \mathrm{Im}\ \rho^2_{10} = \frac{1}{N} \mathrm{Re} \sum_{\lambda_\gamma,\lambda,\lambda'} \mathcal{M}_{\substack{1,1 \\ \lambda,\lambda'}} \mathcal{M}^*_{\substack{-1,0 \\ \lambda,\lambda'}}$$

Observables: Moments of Angular distribution

$$H^0(LM) = \frac{1}{2\pi}\int I(\Omega,\Phi)\, d^L_{M0}(\theta)\cos M\phi\, \mathrm{d}\Omega \mathrm{d}\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma}\int I(\Omega,\Phi)\cos 2\Phi\, d^L_{M0}(\theta)\cos M\phi\, \mathrm{d}\Omega \mathrm{d}\Phi$$

$$\mathrm{Im}\, H^2(LM) = \frac{1}{\pi P_\gamma}\int I(\Omega,\Phi)\sin 2\Phi\, d^L_{M0}(\theta)\sin M\phi\, \mathrm{d}\Omega \mathrm{d}\Phi$$

$m' = m - M$

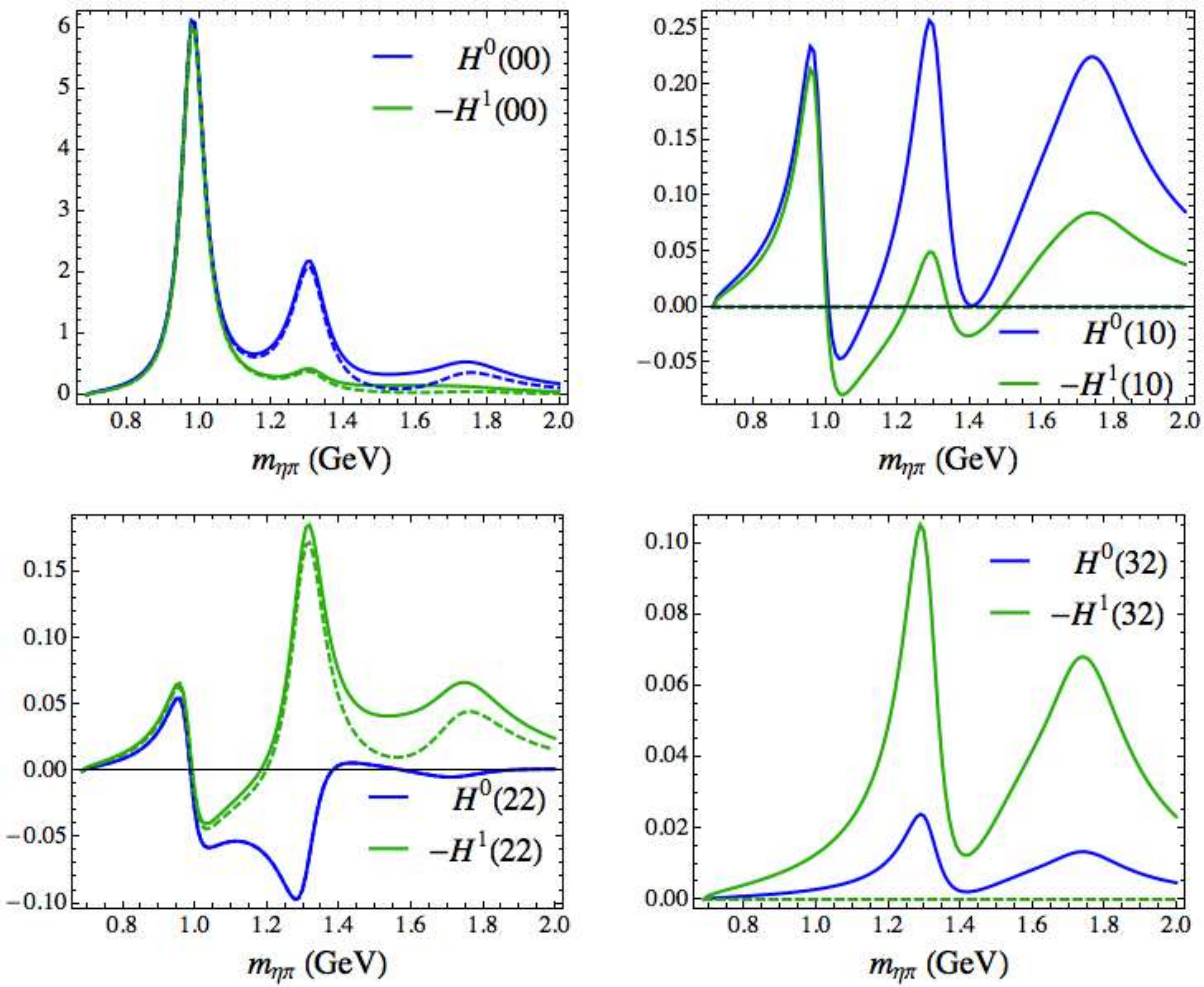
$$H^1(LM) + \mathrm{Im}\ H^2(LM) \propto \sum_{\epsilon,\,\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} \epsilon(-1)^m C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\, [\ell]^{(\epsilon)}_{-m} [\ell']^{(\epsilon)*}_{m'}$$

$0 \leqslant -m\ ;\ 0 \leqslant m'$

The model features only positive projections

$$H^1(LM) + \mathrm{Im}\ H^2(LM) = 0 \qquad M \geqslant 1$$

Moments

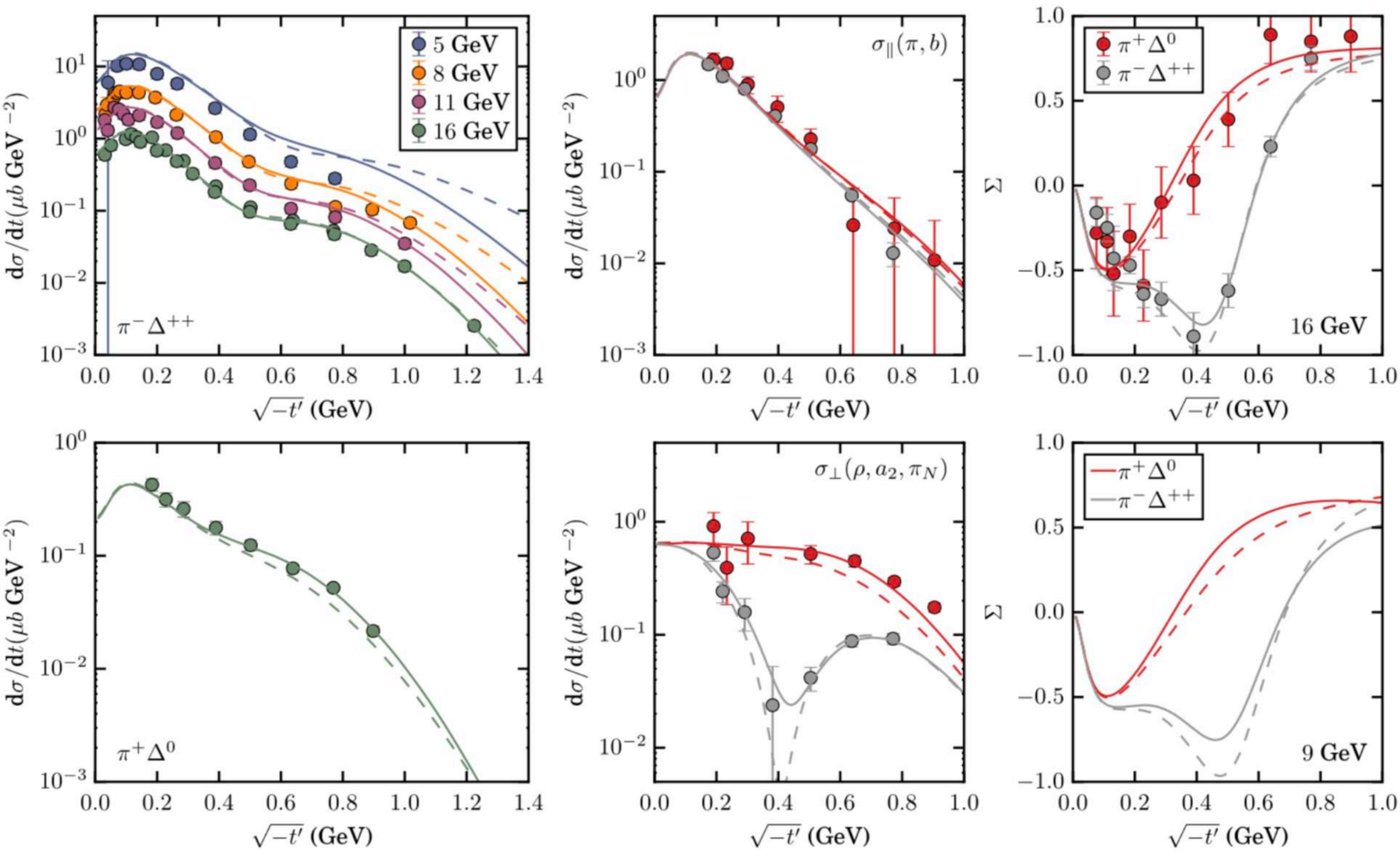


P- wave apparent in odd moments but not in even moments

$a_2(1700)$ more apparent in odd moments than in even moments

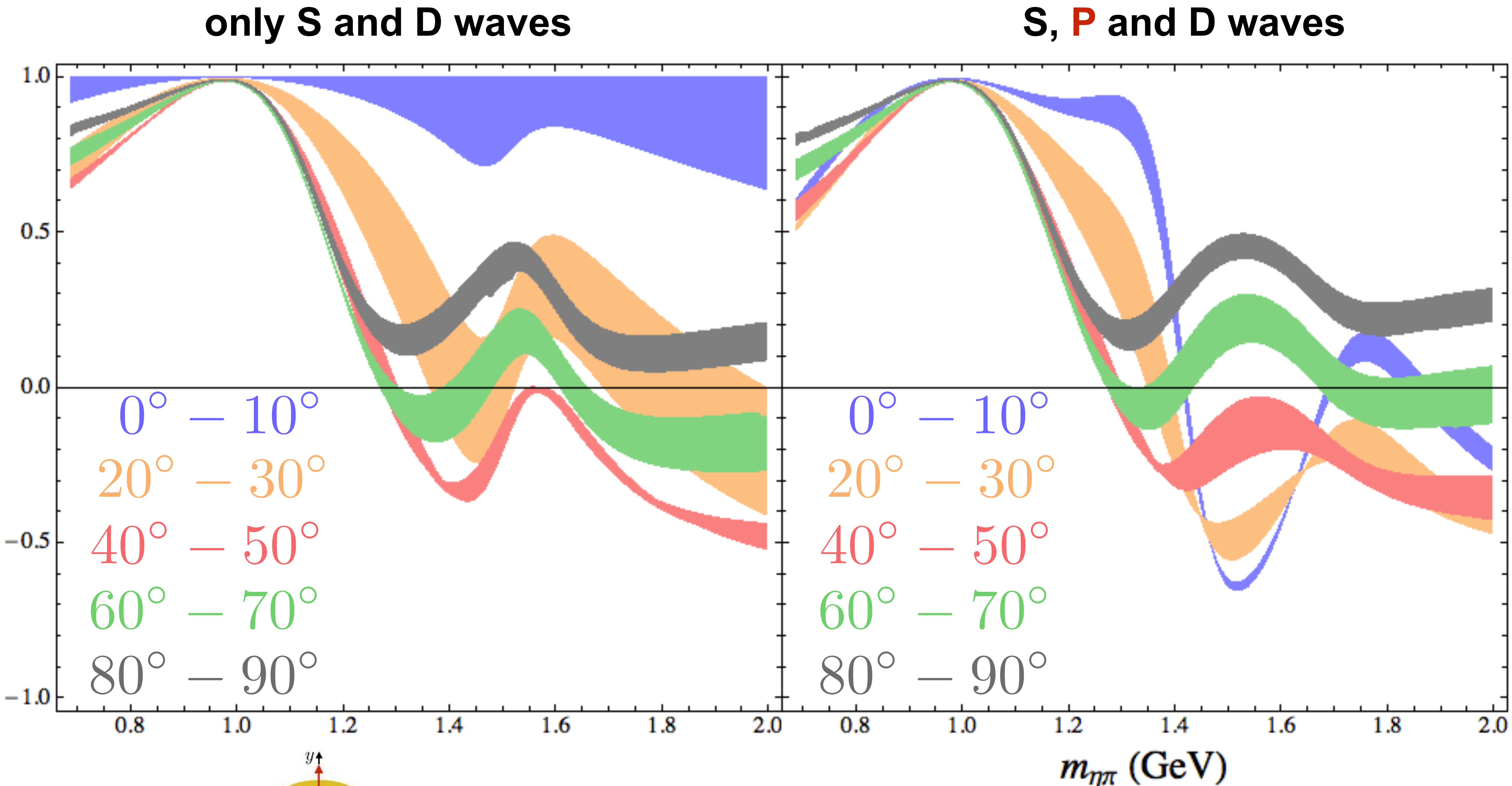
solid lines: *S* + *P* + *D* waves

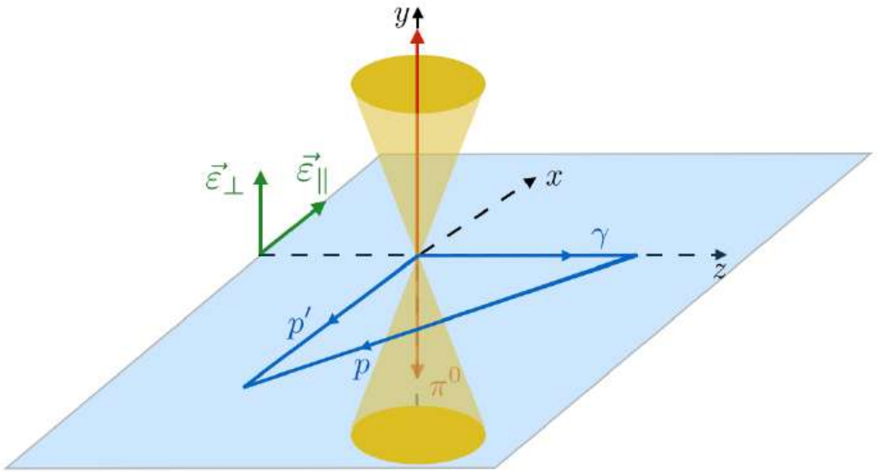
dashed lines: *S* + *D* waves



5 GeV
8 GeV
11 GeV
16 GeV
$\pi^-\Delta^{++}$
$d\sigma/dt(\mu b\ \mathrm{GeV}^{-2})$
$\sqrt{-t'}$ (GeV)
$\sigma_{\parallel}(\pi, b)$
$\pi^+\Delta^0$
$\pi^-\Delta^{++}$
Σ
16 GeV
$\pi^+\Delta^0$
$\sigma_{\perp}(\rho, a_2, \pi_N)$
9 GeV

Beam Asymmetries: $\Sigma_{y\pm\delta^\circ}$





with an opening angle greater than 30° the observables is not sensitive to the P-wave (with our model)

Observables: Moments of Angular distribution

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi)\, d^L_{M0}(\theta) \cos M\phi \, \mathrm{d}\Omega \mathrm{d}\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \cos 2\Phi \, d^L_{M0}(\theta) \cos M\phi \, \mathrm{d}\Omega \mathrm{d}\Phi$$

$$\mathrm{Im}\, H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \sin 2\Phi \, d^L_{M0}(\theta) \sin M\phi \, \mathrm{d}\Omega \mathrm{d}\Phi$$

Moments are unambiguously extracted and are related to partial waves (interferences)

$$^{(+)}H^0(21) = 2\mathrm{Re}\left[\left(\frac{S_0^{(+)}}{\sqrt{5}} + \frac{D_0^{(+)}}{7}\right)\left(D_1^{(+)*} - D_{-1}^{(+)*}\right)\right] + 2\frac{\sqrt{3}}{5}\mathrm{Re}\left[P_0^{(+)}\left(P_1^{(+)*} - P_{-1}^{(+)*}\right)\right]$$
$$+ 2\frac{\sqrt{6}}{7}\mathrm{Re}\left[D_1^{(+)}D_2^{(+)*} - D_{-1}^{(+)}D_{-2}^{(+)*}\right]$$

$$^{(+)}H^1(21) = 2\mathrm{Re}\left[\left(\frac{S_0^{(+)}}{\sqrt{5}} + \frac{D_0^{(+)}}{7}\right)\left(D_1^{(+)*} - D_{-1}^{(+)*}\right)\right] + 2\frac{\sqrt{3}}{5}\mathrm{Re}\left[P_0^{(+)}\left(P_1^{(+)*} - P_{-1}^{(+)*}\right)\right]$$
$$+ 2\frac{\sqrt{6}}{7}\mathrm{Re}\left[D_1^{(+)}D_{-2}^{(+)*} + D_{-1}^{(+)}D_2^{(+)*}\right]$$

$$^{(+)}H^2(21) = -2\mathrm{Re}\left[\left(\frac{S_0^{(+)}}{\sqrt{5}} + \frac{D_0^{(+)}}{7}\right)\left(D_1^{(+)*} + D_{-1}^{(+)*}\right)\right] - 2\frac{\sqrt{3}}{5}\mathrm{Re}\left[P_0^{(+)}\left(P_1^{(+)*} + P_{-1}^{(+)*}\right)\right]$$
$$+ 2\frac{\sqrt{6}}{7}\mathrm{Re}\left[D_1^{(+)}D_{-2}^{(+)*} + D_{-1}^{(+)}D_2^{(+)*}\right]$$