1 Measuring the Charged Pion Polarizability in the $\gamma\gamma \to \pi^+\pi^-$ Reaction

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1.1 Introduction

Electromagnetic polarizabilities are fundamental properties of composite systems such as molecules, atoms, nuclei, and hadrons [Holstein]. Whereas magnetic moments provide information about the ground state properties of a system, polarizabilities provides information about the spectrum of excited states. For atomic systems the polarizabilities are of order the atomic volume. For hadrons the polarizabilities are very much smaller than the volume, typically of order $10^{-4} fm^3$, because of the much greater stiffness of the QCD force as compared to the electromagnetic force. Measurements of hadron polarizabilities provide an important test point for effective field theories, dispersion theories, and lattice calculations. The topic of hadron polarizabilities has remained very compelling, not only because it provides a means to test theory, but also because of the clear l connection with the science of dielectric materials.

Hadron polarizabilities are best measured in Compton scattering experiments, where one looks for a deviation of the cross section from the prediction of Compton scattering from a structureless Dirac particle. The electric and magnetic polarizabilities of the proton, α_p and β_p, have been measured in Compton scattering experiments at Mainz [Mainz], and in the near future it can be expected that neutron polarizabilities can be extracted from Compton scattering experiments on the deuteron and ^3He.

Because a free pion target doesnt exist, the measurements to date of the charged pion polarizability have been plagued by experimental problems, and by theoretical uncertainties in the interpretation of the data. *The charged pion polarizability is probably among the most important tests of low-energy QCD presently unresolved by experiment* [Pa08]. This letter of intent presents a plan to make a new measurement of the charged pion polarizability using the Primakoff process to measure $\gamma\gamma \rightarrow \pi^+\pi^-$ cross sections using the Glue-X detector in Hall D.

1.2 Calculations of the charged pion polarizability

PCAC and leading order $O(p^4)$ chiral perturbation theory (ChPT) both predict that the electric and magnetic polarizabilities of the charged pion (α_π and β_π) are related to the charged pion weak form factors F_V and F_A in $\pi^+ \rightarrow e^+\nu\gamma$.

$$\alpha_\pi = -\beta_\pi \propto \frac{F_A}{F_V} = \frac{1}{6}(L_6 - L_5)$$

where L_5 and L_6 are low energy constants in the Gasser and Leutwyler effective Lagrangian [Ga84] Using recent results from the PIBETA collaboration for F_A and F_V [By09], the $O(p^4)$ ChPT prediction for the charged pion electric and magnetic polarizabilities is given by

$$\alpha_\pi = -\beta_\pi = 2.78 \pm 0.1^{-4} efm^3$$

The $O(p^6)$ corrections are predicted to be relatively small [Bu96,Ga06], giving the following results,

$$\alpha_\pi - \beta_\pi = 5.7 \pm 1.0 \times 10^{-4} efm^3$$

$$\alpha_\pi + \beta_\pi = 0.16 \pm 0.1 \times 10^{-4} efm^3$$

Dispersion relations have also been used to find α_π and β_π, where $\gamma\gamma \rightarrow \pi^+\pi^-$ data are used to fix the dispersion integrals. Fitting $\gamma\gamma \rightarrow \pi^+\pi^-$ data from threshold up to 2.5 GeV, Fil'kov et. al. [Fi06] found that

$$\alpha_\pi - \beta_\pi = 13.0 + 2.6 - 1.9 \times 10^{-4} fm^3$$

$$\alpha_\pi + \beta_\pi = 0.18 + 0.11 - 0.02 \times 10^{-4} fm^3$$

which is in disagreement with ChPT. Pasquini et al. [Pa08] examined the Fil'kov calculation in detail, and noted that the energy extrapolations used by Fil'kov below and above meson resonances leave considerable room for model dependence. When the basic requirements of dispersion relations are taken into account , the predicted vaule from dispersion relations, $\alpha_\pi - \beta_\pi = 5.7 fm^{-4}$, does agree with ChPT [Pa08] .

1.3 Pion polarizability and hadronic light-by-light corrections to $g_\mu - 2$

It is well known that there is a significant difference, 3.6σ , between the E821 experimental value for muon $g_\mu - 2$ and the standard mode (SM) prediction. The errors$(g_\mu - 2)/2$ are approximately 63×10^{-11} for experiment , and 49×10^{-11} for theory. Since the next generation $g_\mu - 2$ experiment at FNAL will reduce the experimental error by a factor of four, it is very important to reduce the SM error by a similar factor. The two largest uncertainties in the SM prediction are from hadronic vacuum polarizaton and hadronic light-by-light (HLBL) scattering. In a recent preprint Ramsey-Musolf and collaborators estimate that an omitted contribution to HLBL from the pion polarizability is substantial and potentially significant [En12]. Work is continuing on this problem.

1.4 Measurements of the charged pion polarizability

Three different experimental techniques that have been utilized to measure α_π and β_π.

- Radiative pion photoproduction, $\gamma p \to \gamma' \pi^+ n$, at very low momentum transfer to the recoil nucleon. This reaction can be visualized as Compton scattering off a virtual pion. At forward Compton angles the reaction is sensitive to $\alpha_\pi + \beta_\pi$, and at backward angles $\alpha_\pi - \beta_\pi$. The most recent measurement has been from Mainz [Ah05] . Using the constraint $\alpha_\pi = -\beta_\pi$ they obtained

  $$\alpha_\pi - \beta_\pi = 11.6 \pm 1.5_{stat} \pm 3.0_{sys} \pm 0.5_{model} \times 10^{-4} fm^3$$

  Combining errors in quadature gives of 3.4 in the standard units, which differs by 1.7σ from the ChPT prediction.

- Primakoff effect of scattering a high energy pion in the Coulomb field of a heavy nucleus, $\pi A \to \pi' \gamma A$. This reaction is equivalent to Comption scattering a nearly real photon off the pion. The most recent measurement has been from Serpukov [An83] . Using the constraint $\alpha_\pi = -\beta_\pi$, they obtained

  $$\alpha_\pi - \beta_\pi = 13.6 \pm 2.8_{stat} \pm 2.4_{sys} \times 10^{-4} fm^3$$

Combining errors in quadrature gives an 3.7 in the standard units, differing by 2.1σ from the ChPT prediction. The COMPASS collaboration at CERN has also taken data, and analysis is underway.

- $\gamma\gamma \to \pi^+\pi^-$. By crossing symmetry (exchanging s and t variables in the scattering amplitude) the $\gamma\gamma \to \pi\pi$ amplitude can be related to the $\gamma\pi \to \gamma\pi$ amplitude. For the $\gamma\gamma \to \pi\pi$ reaction , the sensitivity to the polarizabilities goes as $\alpha_\pi - \beta_\pi$. Babusci et al. [Ba92] used chiral perturbation theory with a one-loop correction to derive a formula they used to obtain pion polarizabilites from $\gamma\gamma \to \pi^+\pi^-$ data. Examining data sets from PLUTO, DM1, DM2, and MARK II , they obtained values of $\alpha_\pi - \beta_\pi$ ranging from 52.6 ± 14.8 (from DM2) to 4.4 ± 3.2 (from MARK II).

It is difficult to draw conclusions from the present experimental results for $\alpha_p i - \beta_\pi$. It is generally recognized that the most model independent technique to measure hadron polarizabilities is through Compton scattering. The two most recent Compton measurements at Serpukov (Primakov) and Mainz (virtual pion) agree that the value for $\alpha_\pi - \beta_\pi$ is approximately twice the size predicted by ChPT, albeit with large errors. New data from Compass are especially welcome to help resolve this descrepanciy.

Turning now to the $\gamma\gamma \to \pi^+\pi^-$ data, the analysis by Babusci [Ba92] was limited by data sets with low statistics (MARKII) and large systematic errors [Mo87]. It was also limited by the calculation, which was only one-loop in ChPT. Since then, considerable theoretical progress has been make in calculating $\gamma\gamma \to \pi\pi$ cross sections; (i) Gasser et al. [Ga06] performed a two-loop calculation in ChPT , (ii) Donoghue and Holstein [Do93] established a connection between dispersiion theory and ChPT by matching the low-energy chiral amplitude with the dispersion treatment, and (iii) Pasquinii et al. [Pa08] performed a purely dispersiive treatment for the cross section.

Fig. 1 shows predicted total cross sections from Pasquini et al. for $\gamma\gamma \to \pi^+\pi^-$ for $|cos\theta_{\pi\pi}| < 0.6$. The red curve is the Born approximation calculation with no polarizability effect. The black solid curve is an unsubtracted dispersion relation (DR) calculation with $\alpha_\pi - \beta_\pi = 5.7$, and the dashed curve is the subtracted DR calculation with the same polarizability. The dotted curve is the subtracted DR calculation with the the polarizabilities from [Fi06] with $\alpha_\pi - \beta_\pi = 13.0$. Comparision of the two subtracted

DR curves with $\alpha_\pi - \beta_\pi$ equal to 5.7 (dashed) and 13.0 (dotted), shows a shift in the peak cross section at $W_{\pi\pi} \approx 0.3$ GeV by approximately 10 percent. We conclude that measurements of the pion polarizability through the $\gamma\gamma \to \pi^+\pi^-$ reaction will need statistical and systematic accuracies at the level of a few percent.

The experimental data in the figure are from MARK-II [Bo92], where there are probably less than 400 events in the region of interest, $W_{\pi\pi} < 0.5$ GeV. The figure clearly shows that the MARK-II data do not have the statistical precison, nor the coverage in $W_{\pi\pi}$, to provide a useful constraint on $\alpha_\pi - \beta_\pi$. It is particularly relevant to quote Donogue and Holstein [Do93] here, "*We conclude that although $\gamma\gamma \to \pi^+\pi^-$ measurements certainly have the potential to provide a precise value for the pion polarizability, the statistical uncertainty of the present values does not allow a particularly precise evaluation.*"

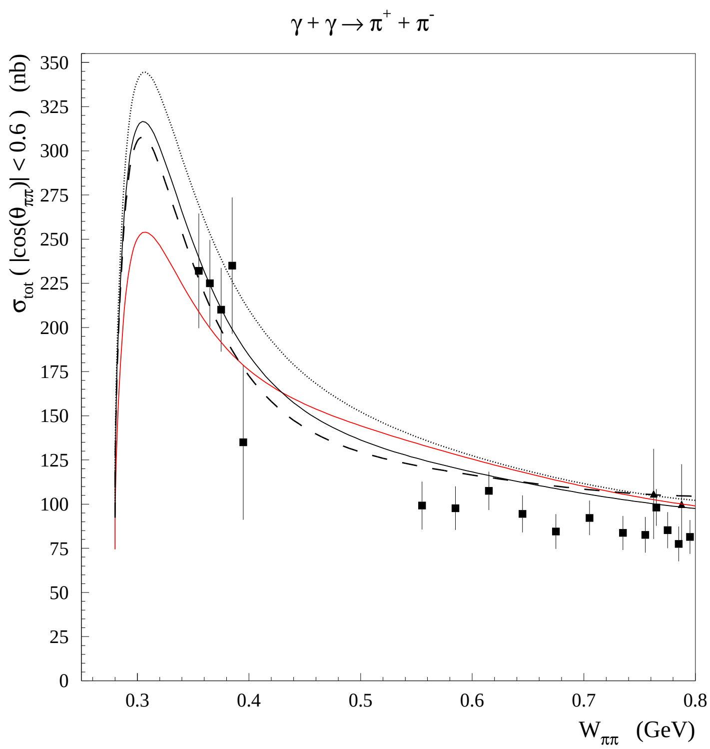

Figure 1: $\gamma\gamma \to \pi^+\pi^-$ cross sections. See text for details of the plot.

1.5 Measurements of the charged pion polarizability at Jefferson Lab Hall D

We propose to make measurements of $\gamma\gamma \to \pi^+\pi^-$ cross sections via the Primakoff effect using the GlueX detector in Hall D. Starting from the Primakoff result for cross sections with incident *linearly polarized photons* [Gl61] and then generalizing for a two-body final state gives,

$$\frac{d^3\sigma}{d\Omega_{\pi\pi}^{Lab} d\Omega_{\pi}^{CM} dW_{\pi\pi}} = \frac{d^2\Gamma(\gamma\gamma \to \pi^+\pi^-)}{d\Omega_{\pi}^{CM} dW_{\pi\pi}} \frac{8\alpha Z^2}{W_{\pi\pi}^3} \frac{\beta^3 E_{\gamma}^4}{Q^4} |F_{EM}(Q)|^2 sin^2\theta_{\pi\pi}(1+P_{\gamma}cos2\phi_{\pi\pi}) \quad (1)$$

where the differential radiative width (rate) for $\gamma\gamma \to \pi^+\pi^-$ is given by

$$\frac{d^2\Gamma(\gamma\gamma \to \pi^+\pi^-)}{d\Omega_{\pi}^{CM} dW_{\pi\pi}} = \frac{d\sigma(\gamma\gamma \to \pi^+\pi^-)}{d\Omega_{\pi}^{CM}} \frac{W_{\pi\pi} k_{\pi}^{CM}}{8\pi^2} \quad (2)$$

In these expressions, $\Omega_{\pi\pi}^{Lab}$ is the lab solid angle for the emission of the $\pi\pi$ system, Ω_{π}^{CM} is solid angle for the emission of the π^+ in the $\pi\pi$ CM frame, $W_{\pi\pi}$ is the $\pi\pi$ invariant mass, Z is the atomic number of the target, β is the velocity of the $\pi\pi$ system, E_{γ} is the energy of the incident photon, $F_{EM}(Q)$ is the electromagnetic form factor for the target with FSI corrections applied, $\theta_{\pi\pi}$ is the lab angle for the $\pi\pi$ system, $\phi_{\pi\pi}$ is the azimuthal angle relative to the incident photon polarizatoin, and k_{π}^{CM} is the momentum of the π^+ in the CM frame. Assuming a 5 percent radiation length lead target, tagged 8.5 GeV photons at a rate of 10^7 photons/s, and a running time of 500 hours, *then approximately 36,000 $\pi^+\pi^-$ Primakov events are produced in the near threshold region up to $W_{\pi\pi} = 0.5$ GeV.*

The largest competing physics background will be from coherent ρ^0 photoproduction. In the s-channel helicity frame (described in Fig. 2) the angular distribution of the pions goes as

$$\frac{3}{8\pi} sin^2\theta_{\pi}(1 + P_{\gamma}cos2\Psi) \quad (3)$$

where Ψ is the azimuthal angle in the heliity frame relative to the photon polariation. By using incident linearly polarized photons from the Hall D photon tagger, and measuring the azimuthal distribution of the $\pi\pi$ system in the lab frame relative to the photon polarization ($\phi_{\pi\pi}$) , and the azimuthal distribution of π^+ in the helicity frame relative to the photon polarization (Ψ) , we estimate that it will be possible to disentangle the contributions from the Primakoff process from the VMD production. The Hall D tagger is planned to have an intensity maximum at 8.5 GeV tagged energy, with a photon polarization of approximately 40 percent at 8.5 GeV.

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