Lepton Pair Production

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Motivation

- An accurate calculation of the muon photoproduction cross section will be useful in normalizing the pion cross section.
- Lepton pair production can be used as a polarimeter for high energy photons.

Multi-photon Exchange Pair Production

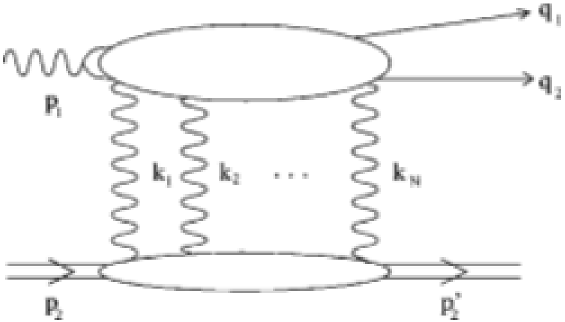

FIG. 1. A diagram with N photons, exchanged in the t channel. Diagrams of this type contribute to the leading asymptotic of lepton pair production by a high energy photon.

Sudakov's Parameterization

Momenta are decomposed into the light-like vectors $k = \omega(1, 1, 0, 0)$, $\tilde{P} = (M/2)(1, -1, 0, 0)$, and two-dimensional vectors transverse to the photon direction.

$$
\begin{aligned}
q &= \alpha_q k + \beta_q \tilde{P} + \mathbf{q} \\
p_1 &= x_1 k + y_1 \tilde{P} + \mathbf{p}_1 \\
p_2 &= x_2 k + y_2 \tilde{P} + \mathbf{p}_2
\end{aligned}
$$

Amplitude

The amplitude for the single photon exchange pair production process can be obtained from the Feynmann rules:

$$M_1 = \frac{ie^3}{q^2}\bar{u}(p_1)\Big[\not{\epsilon}\frac{1}{\not{k}-\not{p}_1-m}\gamma^\mu+\gamma^\mu\frac{1}{\not{k}-\not{p}_2-m}\not{\epsilon}\Big]v(p_2)\big(\bar{\Psi}(P')\gamma_\mu\Psi(P)\big)$$

The result for the N photon exhange is

$$M_N = -i^N s\frac{8\pi^2(eZ)^N}{N!}\int\prod_{i=1}^{N}\frac{\mathrm{d}^2q_i}{(2\pi)^2}\frac{F(q_i^2)}{\mathbf{q}_i^2}\delta^{(2)}(\Sigma q_i - q)J^{(N)}_{\gamma\to ll'}$$

The factor $J^{(N)}_{\gamma\to ll'}$ is called the impact factor:

$$J^{(N)}_{\gamma\to ll'}(\mathbf{p}_1,\mathbf{p}_2) = \bar{u}(p_1)\Big[mS^{(N)}\hat{\epsilon}-2x_1\overrightarrow{T}^{(N)}\overrightarrow{\epsilon}-\hat{T}^{(N)}\hat{\epsilon}\Big]\frac{\hat{\hat{P}}}{s}v(p_2)$$

The functions $S^{(N)}$ and $\overrightarrow{T}^{(N)}$ satisfy recursion relations. From $J^{(N)}_{\gamma\to ll'}$ we form the following functions

$$
\begin{aligned}
J_S^{(N)} &= \int \prod_{i=1}^{N} \frac{\mathrm{d}^2 q_i F(q_i^2)}{\mathbf{q}_i^2} S^{(N)} \delta^{(2)}(\Sigma \mathbf{q}_i - \mathbf{q}) \\
\overrightarrow{J}_T^{(N)} &= \int \prod_{i=1}^{N} \frac{\mathrm{d}^2 q_i F(q_i^2)}{\mathbf{q}_i^2} \overrightarrow{T}^{(N)} \delta^{(2)}(\Sigma \mathbf{q}_i - \mathbf{q})
\end{aligned}
$$

The recursion relations depend on the function

$$
\phi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\pi} \int (\mathrm{e}^{iq\cdot r_2} - \mathrm{e}^{iq\cdot r_1}) \frac{\mathrm{d}^2 q F(q^2)}{\mathbf{q}^2}
$$

Form Factors

For a point-like Coloumb nucleus, $F(q^2) = 1$ and $\phi^c = ln(\frac{\mathbf{r}_1^2}{\mathbf{r}_2^2})$.

Including the atomic screening by the Moliere approximation

$$\frac{F(q^2)}{\mathbf{q}^2} = \frac{1 - F_A}{\mathbf{q}^2} = \sum_{i=1}^{3} \frac{\alpha_i}{\mu_i^2 + \mathbf{q}^2}$$

$$\phi(\mathbf{r}_1, \mathbf{r}_2) = 2 \sum_{i=1}^{3} \alpha_i (K_0(\mu_i |\mathbf{r}_2|) - K_0(\mu_i |\mathbf{r}_1|))$$

Also the nuclear charge form factor can be included $F_N(q^2) = \frac{1}{6} q^2 < r^2 >_A$, where $< r^2 >_A$ is the mean square radius of the nucleus.

Cross Section

The resulting cross section is

$$\begin{aligned} \mathrm{d}\sigma &= \frac{2\alpha\nu^2}{\pi^2}[W_{unp} + \xi_3 W_{pol} cos(2\varphi)]\mathrm{d}x\ \mathrm{d}^2 p_1 \mathrm{d}^2 p_2 \\ W_{unp} &= [x^2 + (1-x)^2]|\overrightarrow{J}_T|^2 + m^2 |J_s|^2 \\ W_{pol} &= -2x(1-x)|\overrightarrow{J}_T|^2 \end{aligned}$$

To determine the cross section, the following functions will be numerically calculated:

$$\begin{aligned} J_s(\mathbf{p}_1, \mathbf{p}_2) &= \frac{i}{2\nu}\int \frac{\mathrm{d}^2 r_1 \mathrm{d}^2 r_2}{(2\pi)^2} \mathrm{e}^{-i(p_1\cdot r_1 + p_2 \cdot r_2)} K_0(m|\mathbf{r}_1 - \mathbf{r}_2|)\nu[\mathrm{e}^{-i\nu\phi} - 1] \\ \overrightarrow{J}_T(\mathbf{p}_1, \mathbf{p}_2) &= \frac{-1}{2\nu}\int \frac{\mathrm{d}^2 r_1 \mathrm{d}^2 r_2}{(2\pi)^2} \mathrm{e}^{-i(p_1\cdot r_1 + p_2 \cdot r_2)} \frac{m(\mathbf{r}_1 - \mathbf{r}_2)}{2|\mathbf{r}_1 - \mathbf{r}_2|} K_1(m|\mathbf{r}_1 - \mathbf{r}_2|)[\mathrm{e}^{-i\nu\phi} - 1] \end{aligned}$$

Algorithm

- Break $J_S(\mathbf{p}_1, \mathbf{p}_2)$ into its real and imaginary parts:

$$
\begin{aligned}
ReJ_S &= \frac{1}{8\pi}\int \mathrm{d}^2r_1\mathrm{d}^2r_2K_0(m|r_1 - r_2|)(sin\alpha(cos\nu\phi - 1) + cos\alpha sin\nu\phi) \\
ImJ_s &= \frac{1}{8\pi}\int \mathrm{d}^2r_1\mathrm{d}^2r_2K_0(m|r_1 - r_2|)(cos\alpha(cos\nu\phi - 1) - sin\alpha sin\nu\phi)
\end{aligned}
$$

- Select values of $\mathbf{p}_1$ and $\mathbf{p}_2$
- for $(lowerLimit < i, j, k, l < upperLimit)$
  Compute $\alpha = \mathbf{p}_1 \cdot r_1 + \mathbf{p}_2 \cdot r_2$
  Get values of K_0 from a look-up table
  $ReJ_s += (\Delta r)^4 * ...$

References

- S. Bakmaev et al. Physics Letters B 660, 494 (2008).
- A. Korchin. Kharkov Institute of Physics and Technology, Ukraine.
- D. Ivanov and K. Melnikov. Phys. Rev. D 57, 4025 (1998).