Theoretical development on $\gamma N \rightarrow mm'N$

Meeting on two mesons photoproduction @ JLab
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Reactions we want to model

Non-strange mesons

- **No charge exchange**
  - $\gamma p \rightarrow \pi^+\pi^- p$ /$f_0(500)$,$\rho(770)$,$f_0(980)$,$f_2(1270)$,heavier f_0's ??,$\rho_3(1690)$,.../
  - $\gamma p \rightarrow \pi^0\pi^0 p$ /$f_0(500)$,$f_0(980)$,$f_2(1270)$,heavier f_0's ??,.../
  - $\gamma p \rightarrow \pi^0\eta p$ /$a_0(980)$,$a_2(1320)$,$a_0(1450)$,$\pi_1(1400)$?/
  - $\gamma p \rightarrow \pi^0\eta' p$ /.../
- **Charge exchange**
  - $\gamma p \rightarrow \pi^+\pi^0 n$ /$\rho(770)$,.../
  - $\gamma p \rightarrow \pi^+\eta n$ /$a_0(980)$,$a_2(1320)$,.../

Reactions we want to model

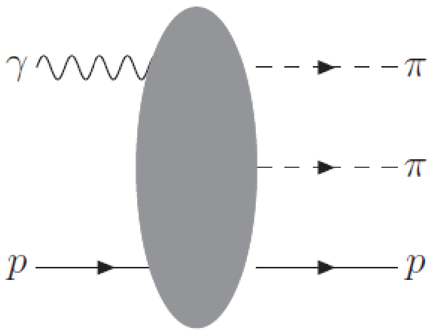
Strange mesons

- **No charge/strangeness exchange**
- $\gamma p \rightarrow K^+K^-(K_0\overline{K}_0)p$ /f_0(980),a_0(980),ϕ(1020),f_2(1270),heavier f_0's ??,ρ_3(1690),.../
- **Strangeness exchange**
- $\gamma p \rightarrow K\pi\Sigma$
- …

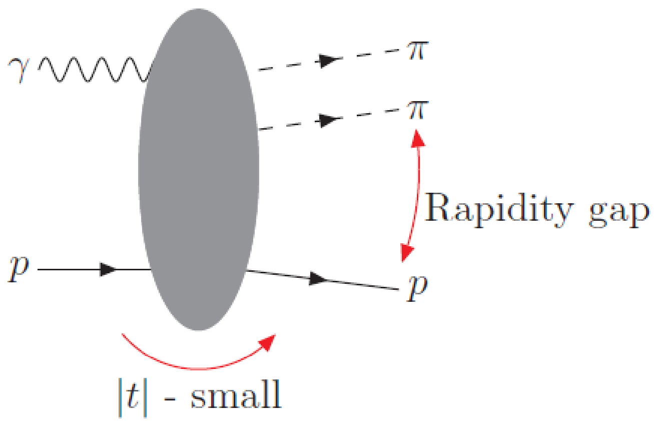
The following discussion is focused on $\pi\pi$ photoproduction but the formalism applicable to any processes where OPE is involved

Kinematics of interest

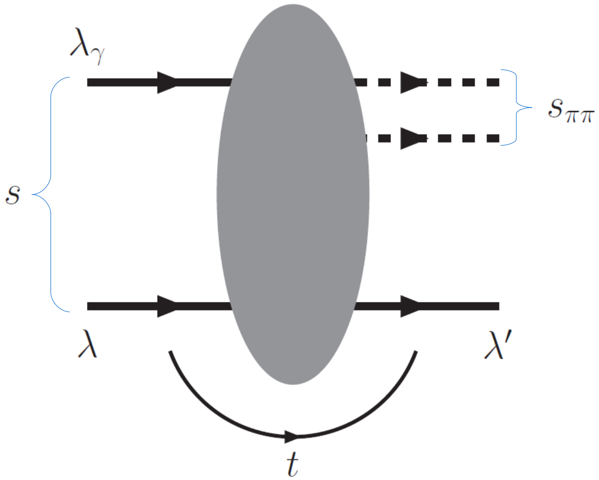
- We are interested in ππp final states where:



- Momentum transfer from target to recoil proton is small
- Such kinematics favors the production of resonances in the ππ system



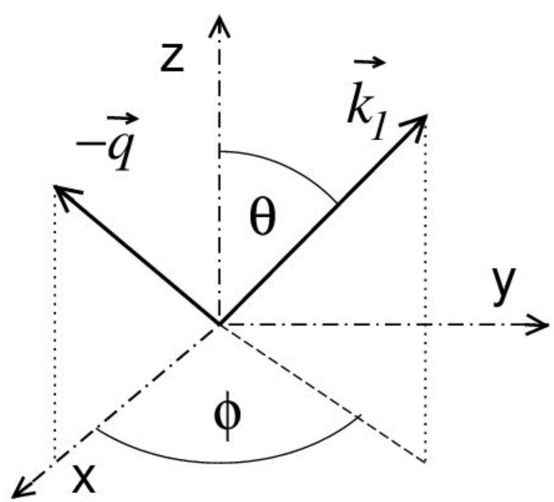
General description of the 3-particle production



The system is described in terms of 5 kinematic variables:

- 3 Lorentz invariants – s, $s_{\pi\pi}$, t
- φ, θ – angles, which describe the outgoing pions momenta (in their CM system), with the z-axis directed opposite to the recoil proton momentum (helicity system)
- and 3 helicities

Definition of the frame of reference



$\hat{z}$ is opposite to recoil proton momentum

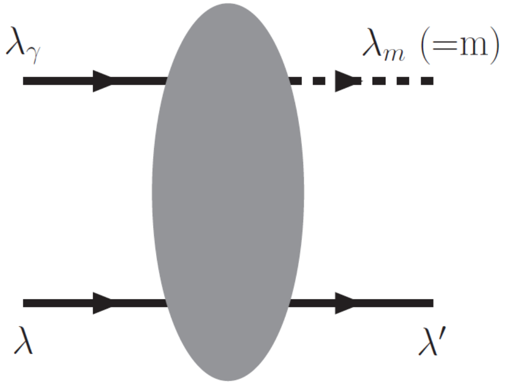
$\hat{y}$ is perpendicular to production plane defined by photon and recoil proton

$$\hat{x} = \hat{y} \times \hat{z}$$

We analyze the ππ system with the following properties:

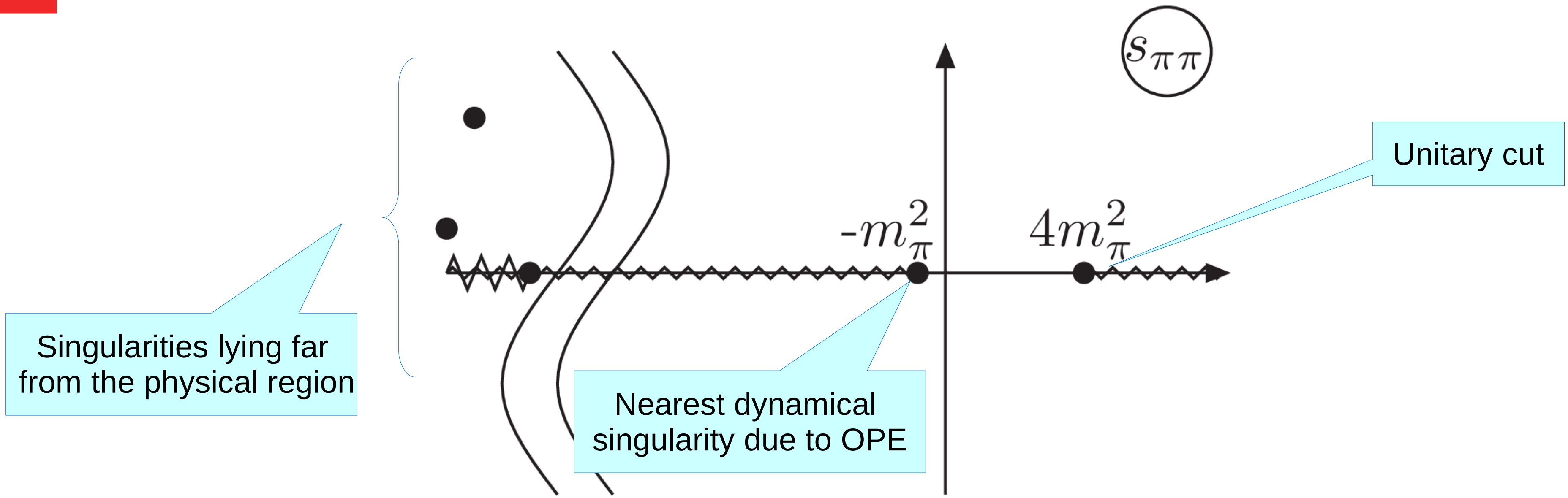
- Total CM energy $\sqrt{s}$ is large in hadronic scale (~10 GeV)
- Effective mass $\sqrt{s_{\pi\pi}}$ is low – so that partial wave expansion of the amplitude is valid

$$A(s, s_{\pi\pi}, t, \theta, \varphi) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} a^l_m(s, s_{\pi\pi}, t) Y^l_m(\theta, \varphi)$$



- For any given partial wave, we can think about the reaction as of the quasi 2→2 scattering
- For fixed *s, t, λ, λ',λ_m* we can treat the partial wave amplitude as a function of one parameter only, ie. $a_{lm}(s,s_{\pi\pi},t)=a(s_{\pi\pi})$

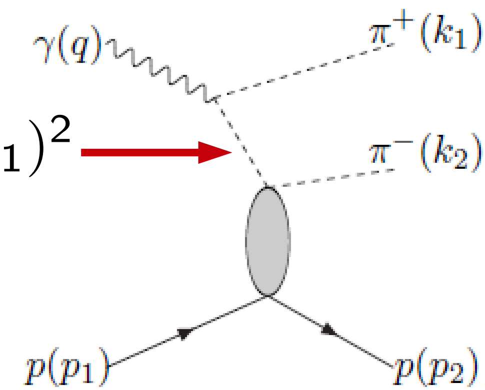
The (approximate) analytical structure of $a(s_{\pi\pi})$:



1. Right hand cut of $a(s_{\pi\pi})$ is determined by unitarity (we neglect coupled channels for a time being)
2. Nearest left hand cut is due to one pion exchange and can be expressed explicitly by Deck amplitude
3. Far away singularities cannot be computed explicitly but can be reliably parameterized, eg. by low degree polynomials in $s_{\pi\pi}$ or modelled (see below)

Diffuse vs compact production source

$t_1 = (q - k_1)^2$



t-channel exchange propagator	Fourier transform of propagator
$1/(t_1 - m^2)$	$\sim \frac{e^{-mr}}{r}$

- Photoproduction of a meson pair through the exchange of:
  - Light particle (or near singularity) <==>diffuse production region
  - Heavy particle (or distant singularity)<==>compact production region
- General form of the amplitude compatible with unitarity (Aitchinson, Bowler 1978) :

$$M = M_{diffuse} e^{i\delta_{\pi\pi}} \cos\delta_{\pi\pi} + M_{compact} e^{i\delta_{\pi\pi}} \sin\delta_{\pi\pi}$$

where:

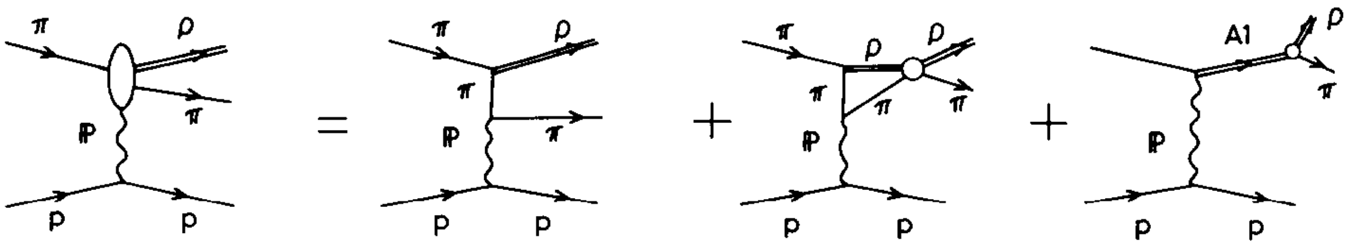
$M_{diffuse}$ – one pion exchange (Deck) amplitude component

$M_{compact}$ – compact source component parameterized as:

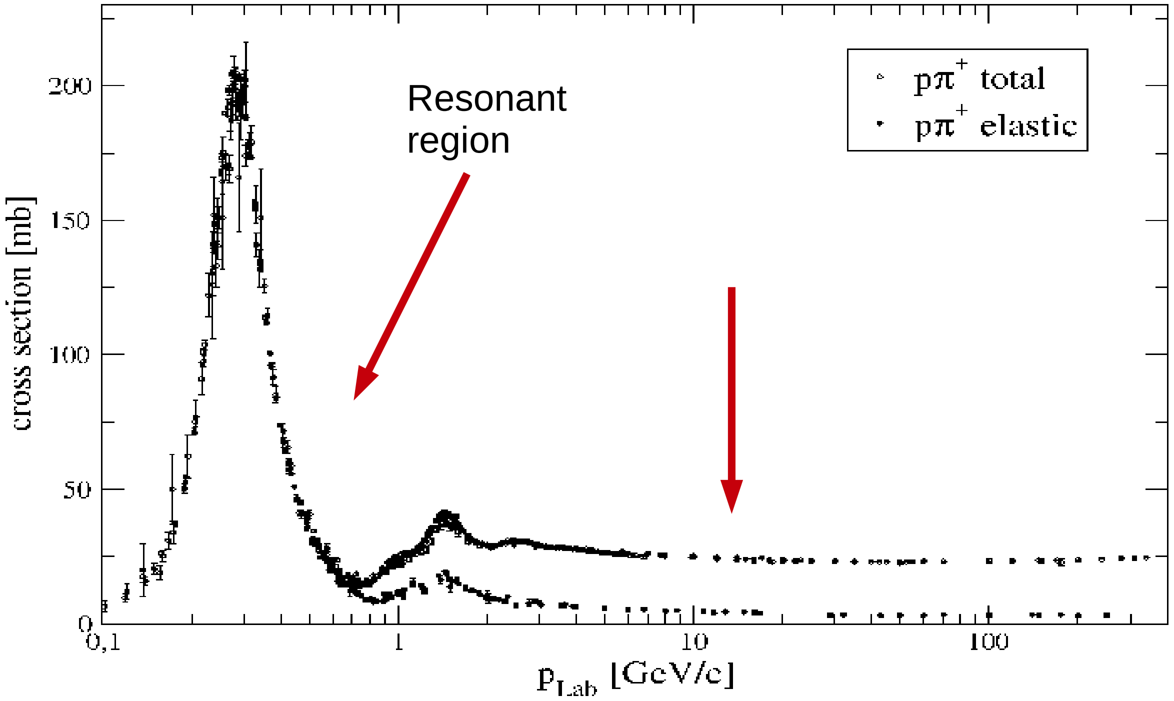
$$\mathbf{M_{compact}} = \boldsymbol{A} + \boldsymbol{B}\ \boldsymbol{s_{\pi\pi}}$$

Generalization of the Deck amplitude

In early versions of the Deck model the πp interaction was assumed to be diffractive



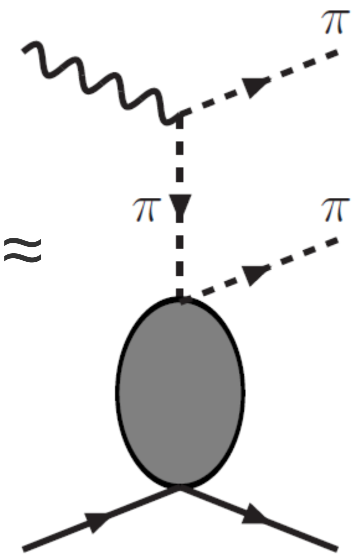
Real experiments, however, cover both diffractive and resonant regimes of πp scattering



Generalization of the Deck amplitude

… so we generalized the Deck amplitude by using SAID partial wave amplitudes which cover both the resonant and diffractive regions up to πp energy of 2.8 GeV

$a_L(s_{\pi\pi}) \approx$

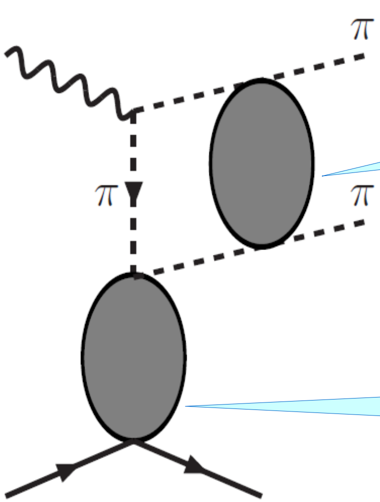


Part of the amplitude dominated by the nearest left hand cut singularity – pion exchange

SAID parametrization of the elastic $\pi p \to \pi p$ amplitude

Important: Such Deck amplitude is basically parameter free !

Rescattering efects or meson resonances produced in the final state



ππ FSI parametrized using dispersion amplitudes by Bydzovsky et al. (Phys.Rev. D94 (2016) 11601)

πp diffraction *and* I=1/2 and I=3/2 baryon resonances N* and Δ are encoded in SAID amplitudes

πp → πp amplitude – partial wave expansion

- General form of the πp scattering amplitude (Chew, Goldberger, Low, Nambu (1957))

$$\mathcal{T}_{\alpha\beta} = \overline{u}(p_2)(A_{\alpha\beta} + \gamma \cdot Q B_{\alpha\beta}) u(p_1)$$

Where: $Q = \frac{1}{2}(q - k_1 + k_2)$ and

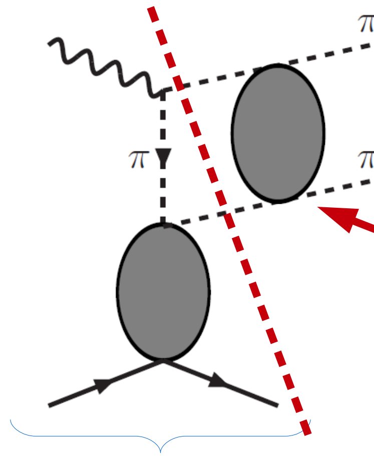
$$\begin{aligned} \frac{A}{4\pi} &= \frac{W+m}{E+m} f_1 - \frac{W-m}{E-m} f_2, \\ \frac{B}{4\pi} &= \frac{f_1}{E+m} + \frac{f_2}{E+m}. \end{aligned}$$

Then the f_1 and f_2 functions are partial wave expanded (separately for I=1/2 and I=3/2):

$$\begin{aligned} f_1 &= \sum_{l=0}^{\infty} f_{l+} P'_{l+1}(\cos\theta^*) - \sum_{l=2}^{\infty} f_{l-} P'_{l-1}(\cos\theta^*), \\ f_2 &= \sum_{l=1}^{\infty} (f_{l-} - f_{l+}) P'_{l}(\cos\theta^*), \end{aligned}$$

f_{l-} and f_{l+} are the functions parameterized by SAID as functions of $s_{\pi p}$.

Translating diagrams into amplitude structure



Initial state amplitude (Deck type amplitude)

$$A_{\pi\pi} = M_{\pi\pi} + \langle \pi\pi | \hat{t}_{FSI} | m'n' \rangle G_{m'n'}(\kappa') M_{m'n'}$$

or in the explicit, partial wave projected form

$$\mathcal{T}^{lm}_{\pi^+\pi^-}(\lambda_2\,\lambda\,\lambda_1) = \left[1 + i\rho\left(\frac{2}{3}t^0_l + \frac{1}{3}t^2_l\right)\right]\mathcal{M}^{lm}_{\pi^+\pi^-}(\lambda_2\,\lambda\,\lambda_1)$$

-even partial waves

$$\mathcal{T}^{lm}_{\pi^+\pi^-}(\lambda_2\,\lambda\,\lambda_1) = \left[1 + i\rho\, t^1_l\right]\mathcal{M}^{lm}_{\pi^+\pi^-}(\lambda_2\,\lambda\,\lambda_1).$$

-odd partial waves

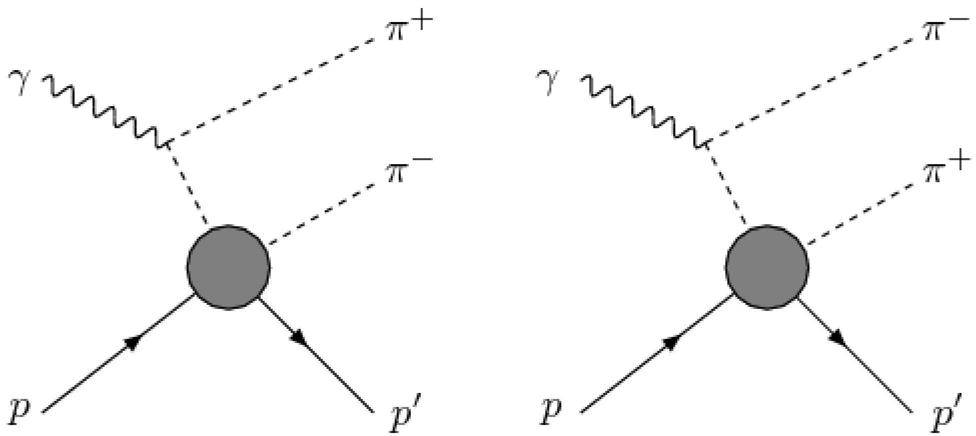
where:

$T^{lm}_{\pi+\pi-}$ – partial wave projected photoproduction amplitude of the meson pair $\pi\pi$,

$M^{lm}_{\pi\pi}$- partial wave projected Deck amplitude,

t^I_l -rescattering amplitude for isospin I and spin l.

Implementing the e-m current conservation in the Deck amplitude

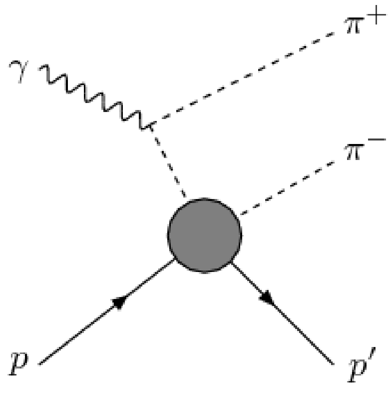


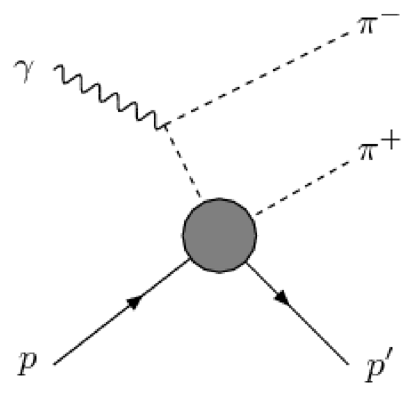
- **General form of the amplitude** [Pumplin 1970]

$$\mathcal{M}_{\lambda_2\lambda_1} = \frac{-1}{\sqrt{4\pi}}\Big\{e\varepsilon\cdot\Big[\frac{\hat{\boldsymbol{\kappa}}}{|\boldsymbol{q}|}\frac{1}{x+\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{\kappa}}}+\frac{\boldsymbol{p_1}+\boldsymbol{p_2}}{q\cdot(p_1+p_2)}\Big]T^{+}{}_{\lambda_2\lambda_1}$$
$$+\,e\varepsilon\cdot\Big[\frac{\hat{\boldsymbol{\kappa}}}{|\boldsymbol{q}|}\frac{1}{x-\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{\kappa}}}-\frac{\boldsymbol{p_1}+\boldsymbol{p_2}}{q\cdot(p_1+p_2)}\Big]T^{-}{}_{\lambda_2\lambda_1}\Big\}$$

- **The amplitude is gauge invariant**

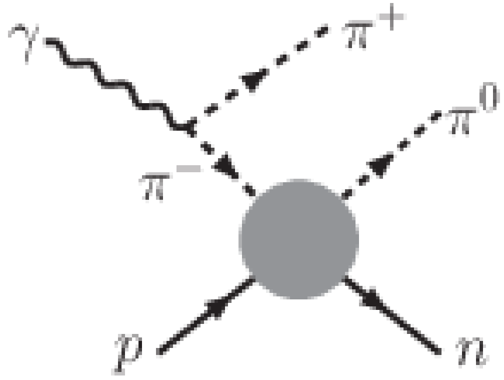
Charge πp amplitudes in terms of isospin amplitudes





$$\mathrm{T}^{-} = \frac{1}{3}(\mathsf{T}^{3/2} + 2\mathsf{T}^{1/2}) \qquad \mathrm{T}^{+} = \mathsf{T}^{3/2}$$

Accordingly...

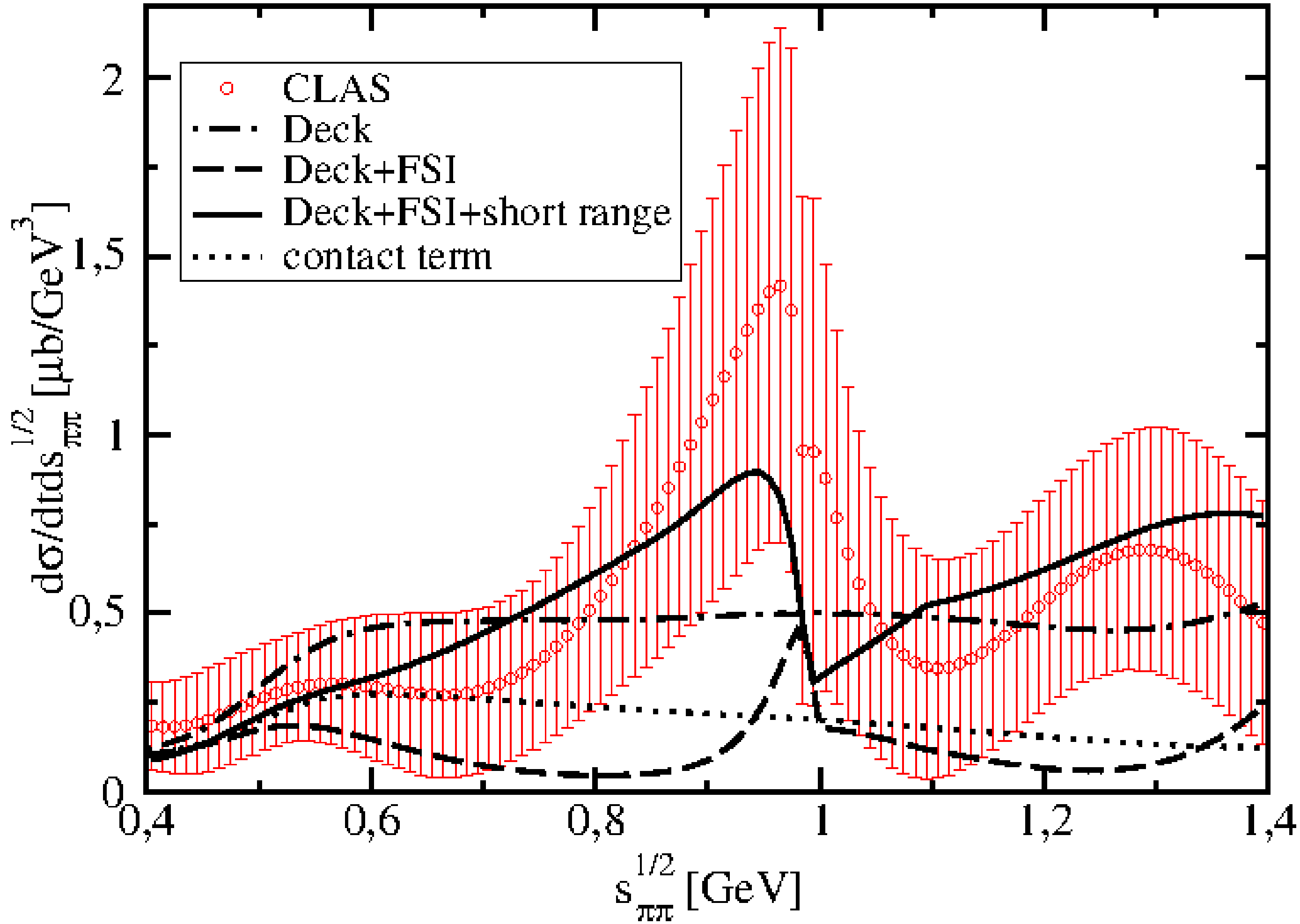


$$\mathrm{T}^{-0} = \sqrt{\frac{2}{3}}(\mathsf{T}^{3/2} - \mathsf{T}^{1/2})$$

Resonances in the $\pi\pi$ partial waves

Mass distributions

S-wave



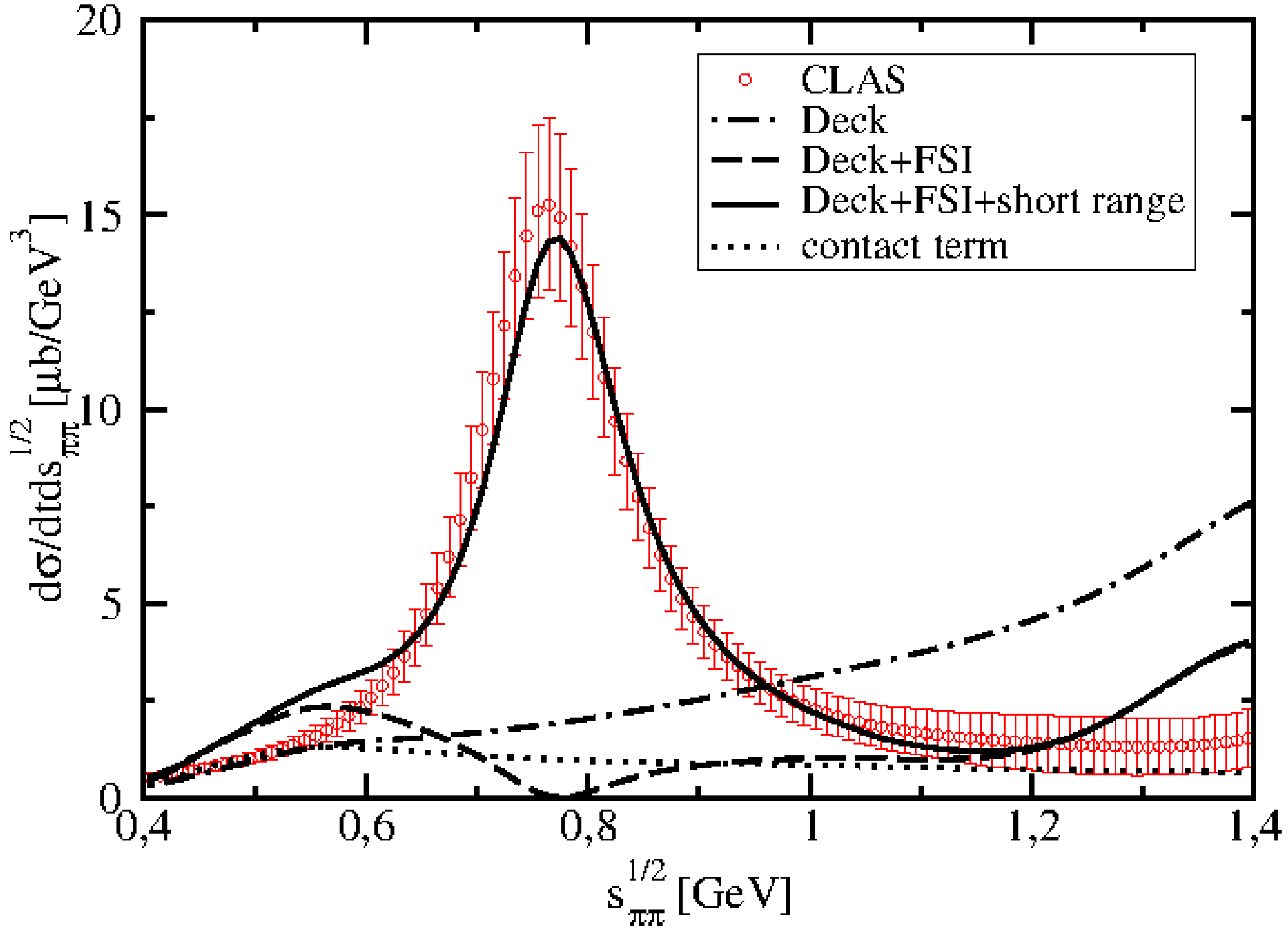
Notes:

- Very good distribution description already at the level of Deck amplitudes
- Clear $f_0(980)$ resonance contribution
- Sizable contribution from the contact term
- Drell+FSI interference is destructive and the theoretical distribution is too small
- Inclusion of the short range component with parameters A=-15 GeV^{-1} and B=3 GeV^{-3} makes the overall fit satisfactory
- Indication of the influence of the coupled $K\bar{K}$ channel above 1 GeV

P-wave

Notes:

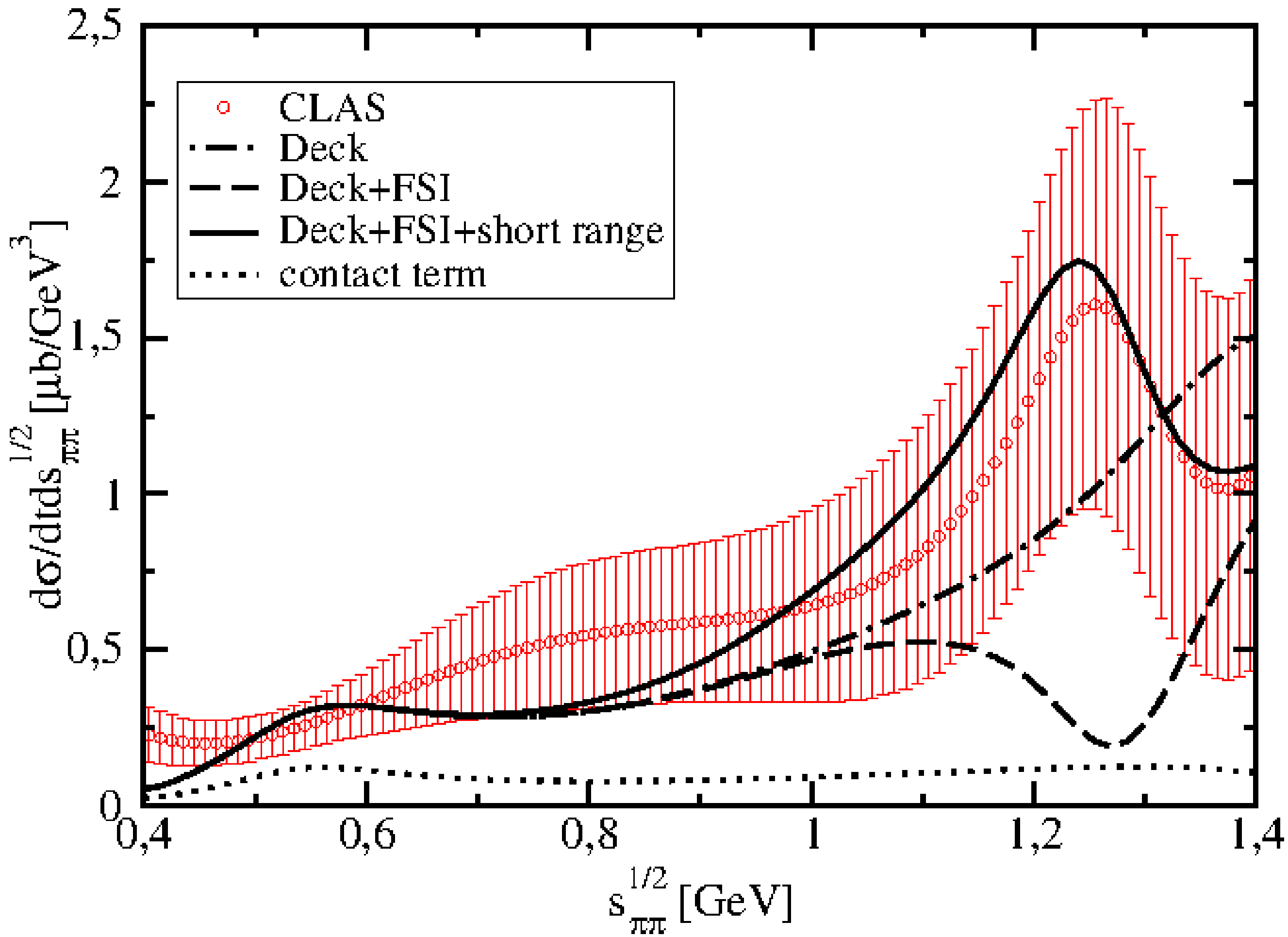
- Very good overall fit to the ρ(770) line
- Deck overshoots the data for masses above 1 GeV but destructive interference with short range component makes the fit better
- Deck+FSI results in minimum rather than maximum at resonance mass
- Fit of the short range component results in good resonance description with parameters: A=49 GeV^{-1} and B=-24 GeV^{-3}



D-wave

Notes:

- Proper magnitude of Deck component
- Deck+FSI results in minimum rather than maximum at resonance mass (same as ρ)
- Fit of the short range component results in good resonance description with parameters: A=-24 GeV^{-1} and B=11 GeV^{-3}



- No indication of the influence of the coupled $K\bar{K}$ channel – quite understandable, $f_2(1270)$ decays to $K\bar{K}$ only in <5% (84% to ππ)
- No additional background needed to describe the data

Hierarchy of the compact component magnitudes

Small short range contribution -
“diffuse source”

Wave	A [GeV^{-1}]	B [GeV^{-3}]
S	−15±1	3±1
P	49±2	−24±2
D	−24±11	10±7

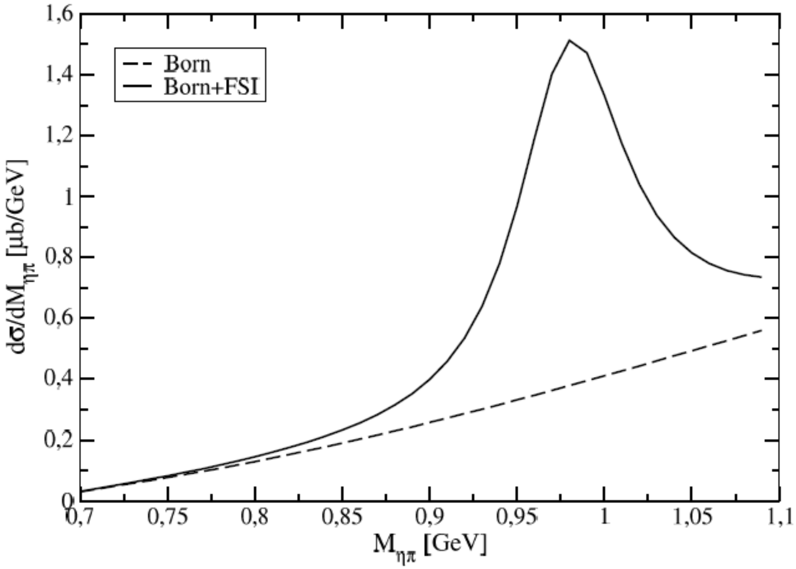
Large short range contribution -
“compact source”

A few observations:

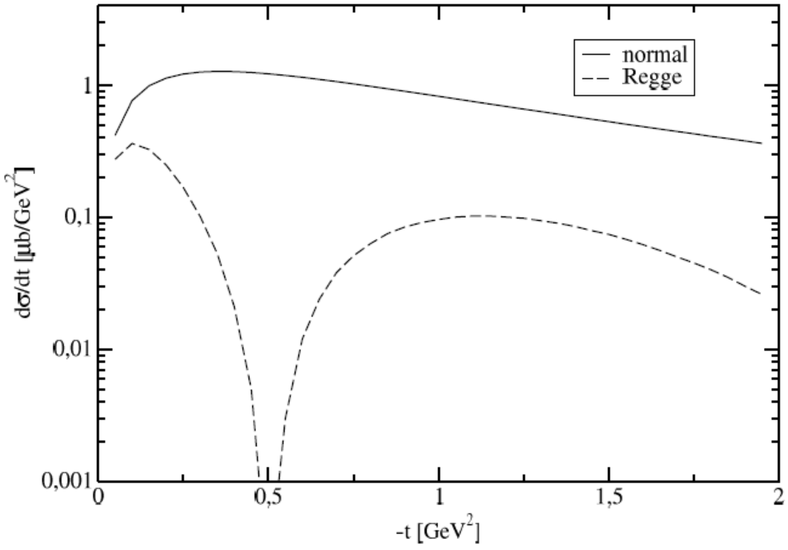
- The model which combines diffuse source (Deck) and compact source components properly describes the *ππ* mass distributions at fixed *t* in *S*-,*P*- and *D*- partial waves and reproduces the dominance of the $f_0(980)$, ρ(770) and $f_2(1270)$ respectively while respecting the 2-particle unitarity in the *ππ* system,
- The relative contribution of the compact source component is large for the ρ(770) and $f_2(1270)$ – in line with the $q\bar{q}$ nature of these resonances,
- The S-wave amplitude is dominated by the diffuse source component, which implies that $f_0(980)$ is a more loosely bound $qq\bar{q}\bar{q}$ object.

πη channel results
/only the short range (vector-vector) exchange/

- S-wave πη mass distribution



- S-wave dσ/dt cross section
-
-
-



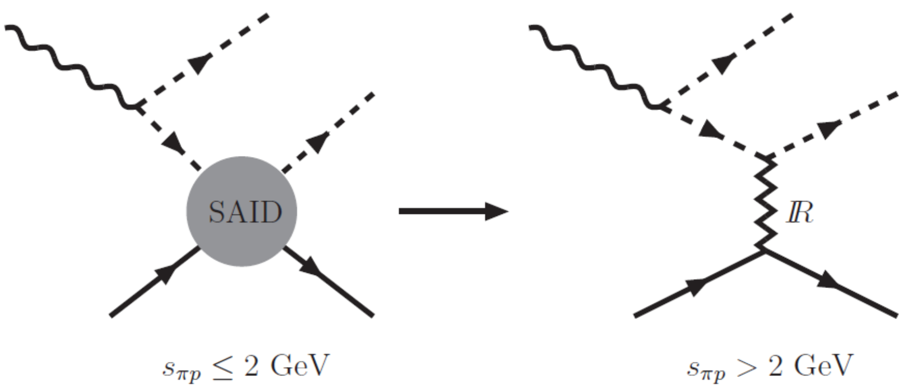
- More on that in: LB, R. Kamiński, Int.J.Mod.Phys. A31 (2016) no.24, 1650139

Work in progress

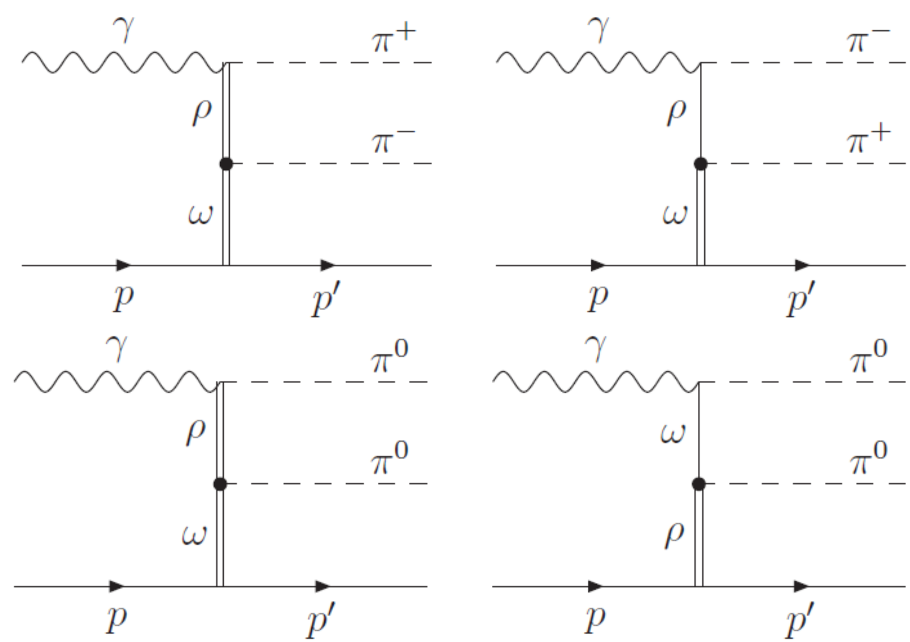
Model for the short range amplitude

- Include the t dependence of the short range part of the amplitude to be able to predict the dσ/dt cross sections
- Short range part should be dominated by the double vector exchange
- So, diagrams to be included in the “Born” amplitude are:

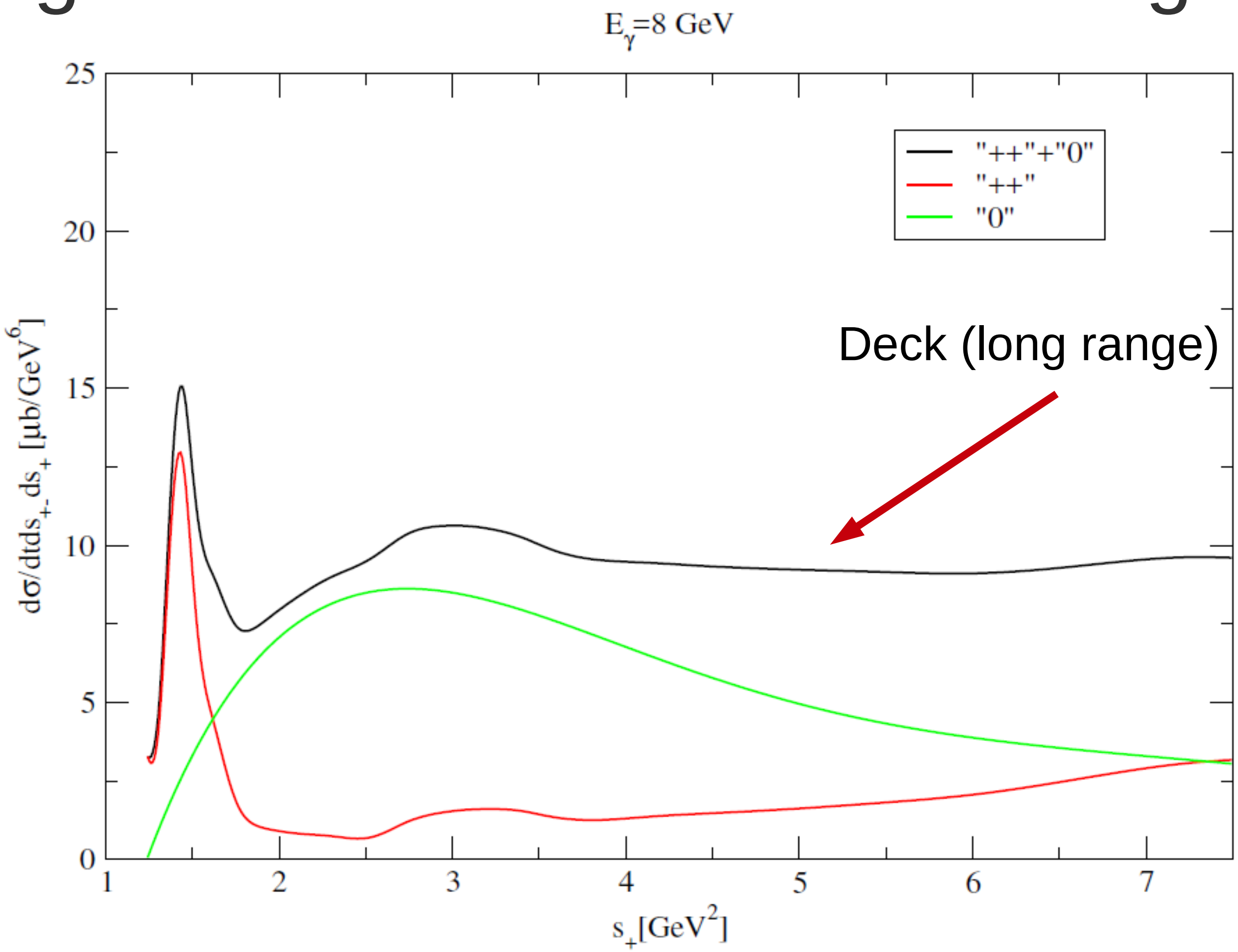
Long range part



Short range part



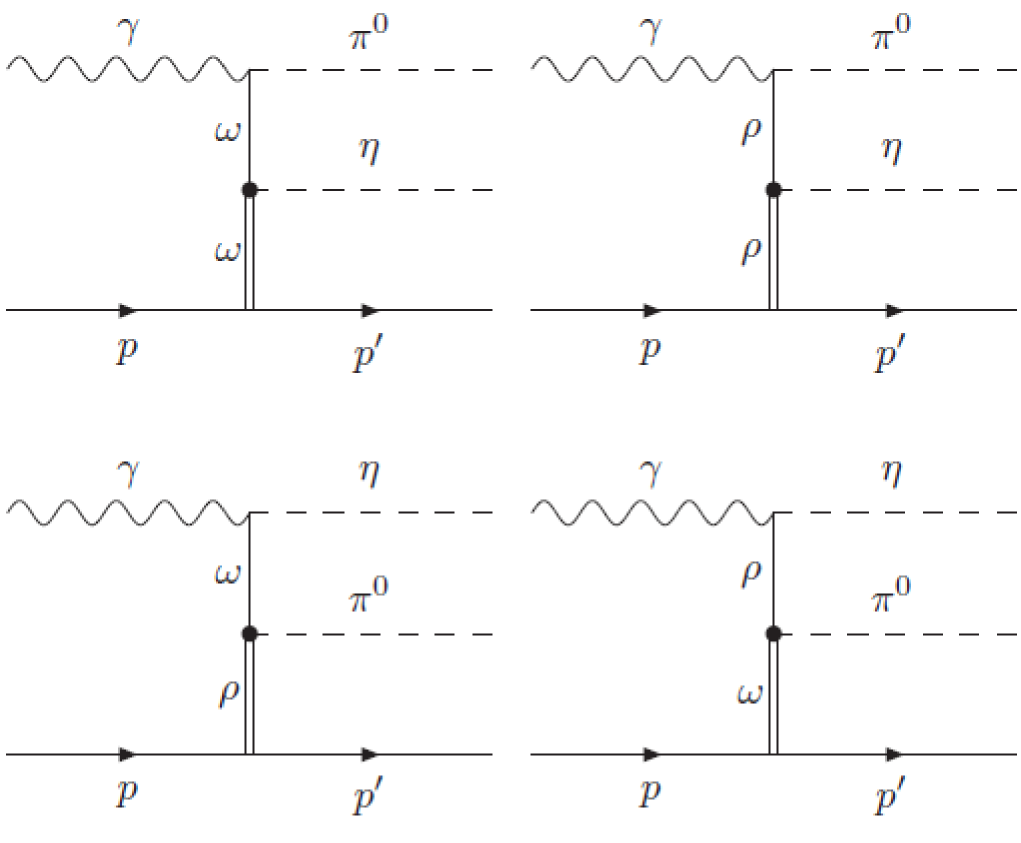
Matching Deck with t-channel exchange at large $s_{p\pi}$



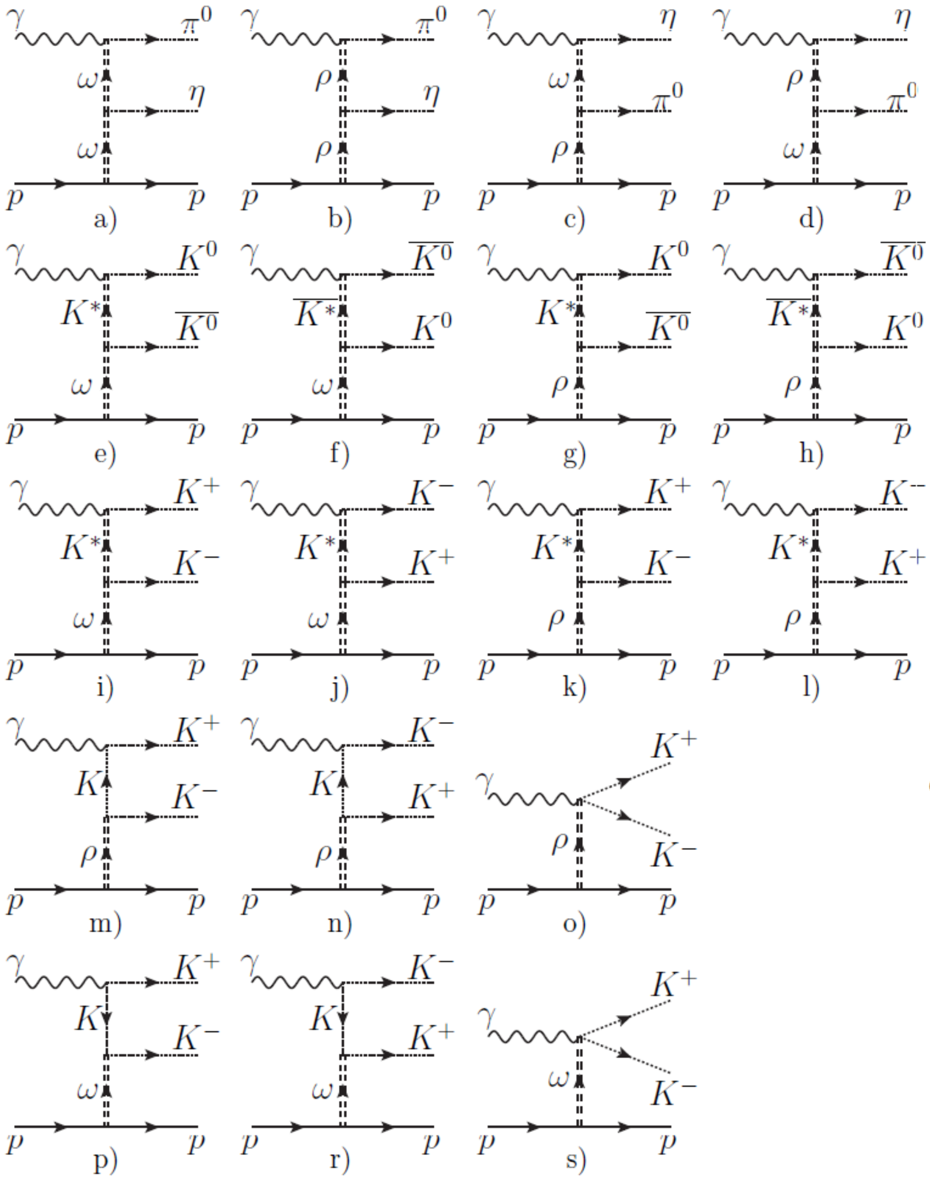
E_γ=8 GeV, t=-0.3 GeV^2, $s_{\pi\pi}$=1 GeV^2

After matching the Deck with the t-channel pion-vector exchange we can model the short range contribution by the vector-vector exchange (couplings are known from radiative decays)

- For doubly neutral pseudoscalar photoproduction, eg. $\pi^0\pi^0$, $\pi^0\eta$ there is no Deck (long range) cotribution
- So, only vector-vector contribute:



Example: Born amplitudes for coupled channel $\pi^0\eta$-KK photoproduction



Two classes of diagrams:

cc'	r=I	r=II
$\pi^0\eta$		(ω,ω), (ρ,ρ), (ρ,ω), (ω,ρ)
$K^0\overline{K^0}$		(K^*,ω), (K^*,ρ)
K^+K^-	(K,ρ), (K,ω)	(K^*,ω), (K^*,ρ)

General form of the photo-amplitude:

$$V_{cc'} = \sum_{r=I,II} \overline{u}(p',s')J_{r,cc'} \cdot \varepsilon(q,\lambda^\gamma)u(p,s),$$

where:

$$J^\mu_{r,cc'} = (\alpha_{r,cc'}g^{\mu\nu} + k_1^\mu \beta^\nu_{1r,cc'} + k_2^\mu \beta^\nu_{2r,cc'}) \times \{d_{r,cc'}\gamma_\nu + e_{r,cc'}(p+p')_\nu\}.$$

For vector-vector (II) unequal cc' masses:

$$\alpha_{II,cc'} = \frac{1}{m_c^2 - m_b^2 - 2q\cdot k_c}\left[-(q\cdot k_{c'})(q\cdot k_c) - (q\cdot k_c)(k_c\cdot k_{c'}) + m_c^2(q\cdot k_{c'})\right],$$

$$\beta_{1II,cc'} = \frac{q(k_c\cdot k_{c'}) - k_c(q\cdot k_{c'})}{m_c^2 - m_b^2 - 2q\cdot k_c},$$

$$\beta_{2II,cc'} = \frac{k_c(q\cdot k_c) - q(m_c^2 - k_c\cdot q)}{m_c^2 - m_b^2 - 2q\cdot k_c}.$$

One can compute all partial waves from this.