Slide for previous meeting

This $\frac{\partial(\text{residual})}{\partial(\text{new track parameters})}$ is necessary for Millepede.

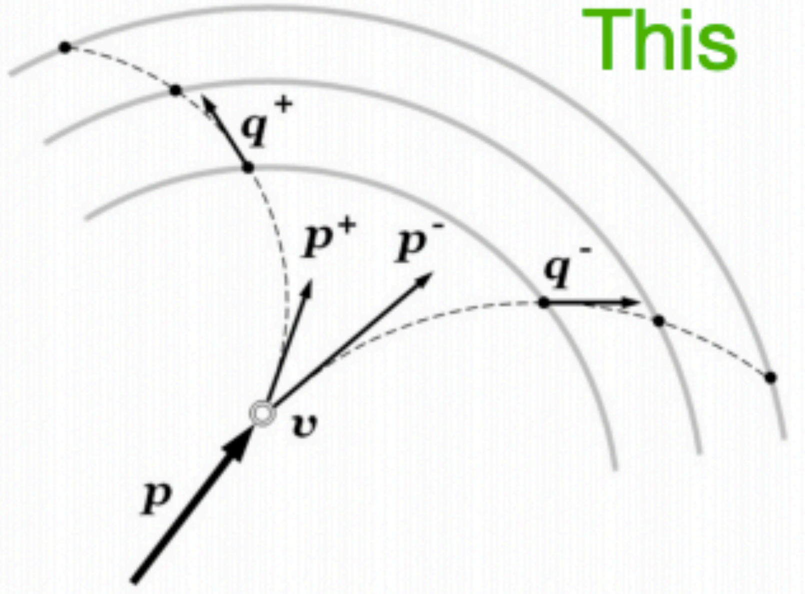

Figure 1: Schematic view of the decay. The dashed lines represent the trajectories, the detector layers are sketched by bold grey lines.

$\frac{\partial(\text{residual})}{\partial(\text{old track parameters})}$, which we already have.

Derivatives of q (old parameters = state vector (x,y,tx,ty,q/p)) with respect to v (Ks decay vertex) and p (pion 3-momentum).

I'm not confident about this part.
—> It's likely we already have these quantities in KinFit part.

4 Representation

The full set of parameters defining the kinematic properties are from now on denoted with $z = (p_x, p_y, p_z, \theta, \phi, M)$. The information fed for instance to an alignment algorithm then takes the following form:

$$\boldsymbol{m} = \begin{pmatrix} \boldsymbol{m}^+ \\ \boldsymbol{m}^- \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}^+(\boldsymbol{v},\boldsymbol{z}) + \boldsymbol{\epsilon}_m^+ \\ \boldsymbol{f}^-(\boldsymbol{v},\boldsymbol{z}) + \boldsymbol{\epsilon}_m^- \end{pmatrix}, \quad \boldsymbol{V}_m = \begin{pmatrix} \boldsymbol{V}_m^+ & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{V}_m^- \end{pmatrix},$$

$$\boldsymbol{D} = \frac{\partial \boldsymbol{f}}{\partial(\boldsymbol{v},\boldsymbol{z})} = \begin{pmatrix} \frac{\partial \boldsymbol{f}^+}{\partial \boldsymbol{q}^+} \cdot \frac{\partial \boldsymbol{q}^+}{\partial \boldsymbol{v}} & \frac{\partial \boldsymbol{f}^+}{\partial \boldsymbol{q}^+} \cdot \frac{\partial \boldsymbol{q}^+}{\partial \boldsymbol{p}^+} \cdot \frac{\partial \boldsymbol{p}^+}{\partial \boldsymbol{z}} \\ \frac{\partial \boldsymbol{f}^-}{\partial \boldsymbol{q}^-} \cdot \frac{\partial \boldsymbol{q}^-}{\partial \boldsymbol{v}} & \frac{\partial \boldsymbol{f}^+}{\partial \boldsymbol{q}^-} \cdot \frac{\partial \boldsymbol{q}^-}{\partial \boldsymbol{p}^-} \cdot \frac{\partial \boldsymbol{p}^-}{\partial \boldsymbol{z}} \end{pmatrix}$$

easy to calculate

Here $\boldsymbol{m}^\pm$ and $\boldsymbol{V}^\pm$ are the measurements and the corresponding covariance matrices of the single trajectories. In the derivative matrix $\boldsymbol{D}$ the chain rule is used, combining the Jacobians $\partial \boldsymbol{f}^\pm/\partial \boldsymbol{q}$ with the Jacobians of the measurement equation $\boldsymbol{q}(\boldsymbol{v}, \boldsymbol{p}_v)$ and the decay model $\boldsymbol{p}^\pm(\boldsymbol{z})$.

3

I couldn't find matrices $\frac{\partial \boldsymbol{q}^+}{\partial \boldsymbol{v}}$ $\frac{\partial \boldsymbol{q}^+}{\partial \boldsymbol{p}^+}$ in the kinematic fitting part,
so I approximately calculate these derivatives assuming that B-field are along z-axis and constant (near the decay vertex).

Eq. of motion

$$\frac{d\boldsymbol{p}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}$$

$$\frac{d\boldsymbol{p}}{dz} = \frac{d\boldsymbol{p}}{dt}\frac{dt}{dz} = \frac{q}{v_z}\boldsymbol{v} \times \boldsymbol{B} = \frac{q}{p_z}\boldsymbol{p} \times \boldsymbol{B} = -\frac{\boldsymbol{p}}{p_z} \times \left(a\hat{\boldsymbol{h}}\right)$$

Assumption

$$\hat{\boldsymbol{h}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

State vector

$$\begin{pmatrix} p_{0x} \\ p_{0y} \\ p_{0z} \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{a(z_0-z)}{p_z}\right) & -\sin\left(\frac{a(z_0-z)}{p_z}\right) & 0 \\ \sin\left(\frac{a(z_0-z)}{p_z}\right) & \cos\left(\frac{a(z_0-z)}{p_z}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x + \frac{p_{0y}-p_y}{a} \\ y - \frac{p_{0x}-p_x}{a} \end{pmatrix} = \begin{pmatrix} x + \frac{\sin\left(\frac{a(z_0-z)}{p_z}\right)p_x + \left(\cos\left(\frac{a(z_0-z)}{p_z}\right)-1\right)p_y}{a} \\ y - \frac{\left(\cos\left(\frac{a(z_0-z)}{p_z}\right)-1\right)p_x - \sin\left(\frac{a(z_0-z)}{p_z}\right)p_y}{a} \end{pmatrix}$$

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} \frac{p_{0x}}{p_{0z}} \\ \frac{p_{0y}}{p_{0z}} \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_x}{p_z} - \sin\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_y}{p_z} \\ \sin\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_x}{p_z} + \cos\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_y}{p_z} \end{pmatrix}$$

Symbol	Description
c	The speed of light in a vacuum.
q	The charge of the particle.
m	The mass of the particle.
E	The energy of the particle.
$\boldsymbol{x}_0$	The measured/reconstructed position of a detected/decaying particle.
$\boldsymbol{x}$	The common vertex to which a group of particles are constrained.
$\Delta\boldsymbol{x}$	$\boldsymbol{x} - \boldsymbol{x}_0$
t_0	The measured/reconstructed time of a detected/decaying particle at $\boldsymbol{x}_0$.
t	The common time to which a group of particles are constrained at position $\boldsymbol{x}$.
Δt	$t - t_0$
$\boldsymbol{p}_0$	The measured/reconstructed momentum of a detected/decaying particle at $\boldsymbol{x}_0$.
$\boldsymbol{p}$	The momentum of a particle at position $\boldsymbol{x}$.
a	$-0.00299792458Bq$
B	The magnitude of the magnetic field at position $\boldsymbol{x}_0$.
$\hat{\boldsymbol{h}}$	The direction unit vector of the magnetic field at position $\boldsymbol{x}_0$.
s_i	The sign for the momentum of particle i used to reconstruct the momentum $\boldsymbol{p}_{0X}$ of a decaying particle X. It's -1 if $\boldsymbol{p}_{0X}$ is defined by missing mass and i is a final-state particle, and is 1 otherwise.

Table 2: Descriptions of the kinematic terms used for constraining the system of particles.

$$\frac{\partial \boldsymbol{q}^+}{\partial \boldsymbol{v}} = \begin{pmatrix} \frac{\partial x}{\partial v_x} & \frac{\partial x}{\partial v_y} & \frac{\partial x}{\partial v_z} \\ \frac{\partial y}{\partial v_x} & \frac{\partial y}{\partial v_y} & \frac{\partial y}{\partial v_z} \\ \frac{\partial t_x}{\partial v_x} & \frac{\partial t_x}{\partial v_y} & \frac{\partial t_x}{\partial v_z} \\ \frac{\partial t_y}{\partial v_x} & \frac{\partial t_y}{\partial v_y} & \frac{\partial t_y}{\partial v_z} \\ \frac{\partial (q/p)}{\partial v_x} & \frac{\partial (q/p)}{\partial v_y} & \frac{\partial (q/p)}{\partial v_z} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\partial \boldsymbol{q}^+}{\partial \boldsymbol{p}^+} = \begin{pmatrix} \frac{\partial x}{\partial p_x^+} & \frac{\partial x}{\partial p_y^+} & \frac{\partial x}{\partial p_z^+} \\ \frac{\partial y}{\partial p_x^+} & \frac{\partial y}{\partial p_y^+} & \frac{\partial y}{\partial p_z^+} \\ \frac{\partial t_x}{\partial p_x^+} & \frac{\partial t_x}{\partial p_y^+} & \frac{\partial t_x}{\partial p_z^+} \\ \frac{\partial t_y}{\partial p_x^+} & \frac{\partial t_y}{\partial p_y^+} & \frac{\partial t_y}{\partial p_z^+} \\ \frac{\partial (q/p)}{\partial p_x^+} & \frac{\partial (q/p)}{\partial p_y^+} & \frac{\partial (q/p)}{\partial p_z^+} \end{pmatrix}$$

$$\frac{\partial x}{\partial p_x^+} = \frac{1}{a}\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{15}$$

$$\frac{\partial x}{\partial p_y^+} = \frac{1}{a}\left(\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) - 1\right) \tag{16}$$

$$\frac{\partial x}{\partial p_z^+} = -\frac{z_0 - z}{p_z^{+2}}\left(p_x^+\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) - p_y^+\sin\left(\frac{a(z_0 - z)}{p_z^+}\right)\right) \tag{17}$$

$$\frac{\partial y}{\partial p_x^+} = -\frac{1}{a}\left(\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) - 1\right) \tag{18}$$

$$\frac{\partial y}{\partial p_y^+} = \frac{1}{a}\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{19}$$

$$\frac{\partial y}{\partial p_z^+} = -\frac{z_0 - z}{p_z^{+2}}\left(p_x^+\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) + p_y^+\cos\left(\frac{a(z_0 - z)}{p_z^+}\right)\right) \tag{20}$$

$$\frac{\partial t_x}{\partial p_x^+} = \frac{1}{p_z^+}\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{21}$$

$$\frac{\partial t_x}{\partial p_y^+} = -\frac{1}{p_z^+}\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{22}$$

$$\frac{\partial t_x}{\partial p_z^+} = \left(\frac{a(z_0 - z)p_y^+}{p_z^{+3}} - \frac{p_x^+}{p_z^{+2}}\right)\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) + \left(\frac{a(z_0 - z)p_x^+}{p_z^{+3}} + \frac{p_y^+}{p_z^{+2}}\right)\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{23}$$

$$\frac{\partial t_y}{\partial p_x^+} = \frac{1}{p_z^+}\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{24}$$

$$\frac{\partial t_y}{\partial p_y^+} = \frac{1}{p_z^+}\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{25}$$

$$\frac{\partial t_y}{\partial p_z^+} = -\left(\frac{a(z_0 - z)p_x^+}{p_z^{+3}} + \frac{p_y^+}{p_z^{+2}}\right)\cos\left(\frac{a(z_0 - z)}{p_z^+}\right) + \left(\frac{a(z_0 - z)p_y^+}{p_z^{+3}} - \frac{p_x^+}{p_z^{+2}}\right)\sin\left(\frac{a(z_0 - z)}{p_z^+}\right) \tag{26}$$

Derivatives for $\frac{\partial \boldsymbol{p}^+}{\partial \boldsymbol{z}}$

$$\frac{\partial}{\partial p_x}\begin{pmatrix} \frac{p_x p_z}{p_T p} & -\frac{p_y}{p_T} & \frac{p_x}{p} \\ \frac{p_y p_z}{p_T p} & \frac{p_x}{p_T} & \frac{p_y}{p} \\ -\frac{p_T}{p} & 0 & \frac{p_z}{p} \end{pmatrix} = \begin{pmatrix} \frac{p_z}{p_T p} - \frac{p_x^2 p_z (p^2+p_T^2)}{p_T^3 p^3} & \frac{p_x p_y}{p_T^3} & \frac{p^2 - p_x^2}{p^3} \\ -\frac{p_x p_y p_z (p^2+p_T^2)}{p_T^3 p^3} & \frac{p_y^2}{p_T^3} & -\frac{p_x p_y}{p^3} \\ -\frac{p_x p_z^2}{p_T p^3} & 0 & -\frac{p_x p_z}{p^3} \end{pmatrix}$$

$$\frac{\partial}{\partial p_y}\begin{pmatrix} \frac{p_x p_z}{p_T p} & -\frac{p_y}{p_T} & \frac{p_x}{p} \\ \frac{p_y p_z}{p_T p} & \frac{p_x}{p_T} & \frac{p_y}{p} \\ -\frac{p_T}{p} & 0 & \frac{p_z}{p} \end{pmatrix} = \begin{pmatrix} -\frac{p_x p_y p_z (p^2+p_T^2)}{p_T^3 p^3} & -\frac{p_x^2}{p_T^3} & -\frac{p_x p_y}{p^3} \\ \frac{p_z}{p_T p} - \frac{p_y^2 p_z (p^2+p_T^2)}{p_T^3 p^3} & -\frac{p_x p_y}{p_T^3} & \frac{p^2 - p_y^2}{p^3} \\ -\frac{p_y p_z^2}{p_T p^3} & 0 & -\frac{p_y p_z}{p^3} \end{pmatrix}$$

$$\frac{\partial}{\partial p_z}\begin{pmatrix} \frac{p_x p_z}{p_T p} & -\frac{p_y}{p_T} & \frac{p_x}{p} \\ \frac{p_y p_z}{p_T p} & \frac{p_x}{p_T} & \frac{p_y}{p} \\ -\frac{p_T}{p} & 0 & \frac{p_z}{p} \end{pmatrix} = \begin{pmatrix} \frac{p_x p_T}{p^3} & 0 & -\frac{p_x p_z}{p^3} \\ \frac{p_y p_T}{p^3} & 0 & -\frac{p_y p_z}{p^3} \\ \frac{p_T p_z}{p^3} & 0 & \frac{p_T^2}{p^3} \end{pmatrix}$$

$$\frac{\partial}{\partial p_x}\left(\frac{p}{2} \pm \frac{1}{2}\sqrt{\frac{\alpha^2 - 1}{\alpha^2}(p^2 + M^2)}\cos\theta\right) = \frac{p_x}{2p} \pm \frac{p_x(\alpha^2 - 1)\cos\theta}{2\alpha^2\sqrt{\frac{\alpha^2-1}{\alpha^2}(p^2 + M^2)}}$$

$$\frac{\partial (q/p)}{\partial p_x^+} = -\frac{q p_x^+}{p^{+3}}$$

$$\frac{\partial (q/p)}{\partial p_y^+} = -\frac{q p_y^+}{p^{+3}}$$

$$\frac{\partial (q/p)}{\partial p_z^+} = -\frac{q p_z^+}{p^{+3}}$$

Using these derivatives, Millepede minimization converges with more than one free parameters.