$\rho(770)$ Meson Spin-Density Matrix Elements

Discussion of Uncertainties

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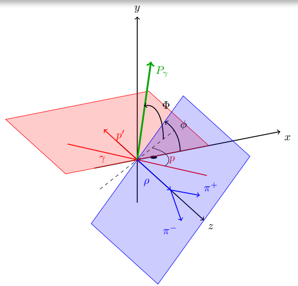
- Full angular distribution of vector meson production and decay is described by **spin-density matrix elements ρ^k_{ij}**
- Linear beam polarization provides access to **nine** linearly independent SDMEs
- Intensity W is expressed as function of angles **$\cos\vartheta, \varphi, \Phi$** and degree of polarization P_γ



$$W(\cos\vartheta,\varphi,\Phi) = W^0(\cos\vartheta,\varphi) - P_\gamma\cos(2\Phi)W^1(\cos\vartheta,\varphi) - P_\gamma\sin(2\Phi)W^2(\cos\vartheta,\varphi)$$

$$W^0(\cos\vartheta,\varphi) = \frac{3}{4\pi}\left(\frac{1}{2}(1-\rho^0_{00}) + \frac{1}{2}(3\rho^0_{00}-1)\cos^2\vartheta - \sqrt{2}\mathrm{Re}\rho^0_{10}\sin 2\vartheta\cos\varphi - \rho^0_{1-1}\sin^2\vartheta\cos 2\varphi\right)$$

$$W^1(\cos\vartheta,\varphi) = \frac{3}{4\pi}\left(\rho^1_{11}\sin^2\vartheta + \rho^1_{00}\cos^2\vartheta - \sqrt{2}\mathrm{Re}\rho^1_{10}\sin 2\vartheta\cos\varphi - \rho^1_{1-1}\sin^2\vartheta\cos 2\varphi\right)$$

$$W^2(\cos\vartheta,\varphi) = \frac{3}{4\pi}\left(\sqrt{2}\mathrm{Im}\rho^2_{10}\sin 2\vartheta\sin\varphi + \mathrm{Im}\rho^2_{1-1}\sin^2\vartheta\sin 2\varphi\right)$$

Schilling *et al.* [Nucl. Phy. B, 15 (1970) 397]

Extraction of SDMEs

$$W(\cos\vartheta,\varphi,\Phi) = W^0(\cos\vartheta,\varphi) - P_\gamma\cos(2\Phi)W^1(\cos\vartheta,\varphi) - P_\gamma\sin(2\Phi)W^2(\cos\vartheta,\varphi)$$

$$\text{Measured Intensity } I(\Omega) \propto W(\cos\vartheta,\varphi,\Phi)$$

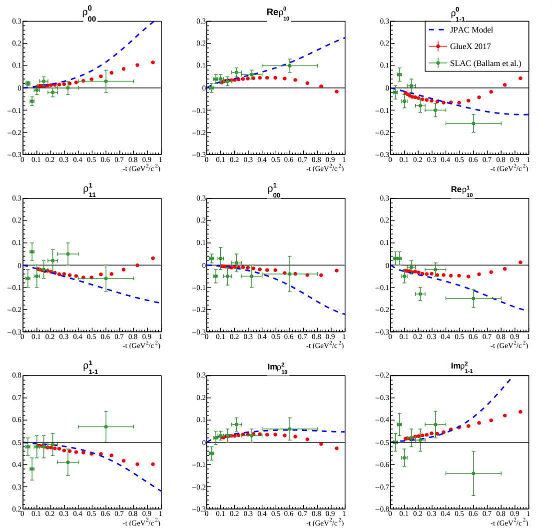
Extended Maximum-Likelihood Fit

$$\ln L = \underbrace{\sum_{i=1}^{N} \ln I(\Omega_i)}_{\text{Signal Events}} - \underbrace{\sum_{j=1}^{M} \ln I(\Omega_j)}_{\text{Background}} - \underbrace{\int d\Omega\, I(\Omega)\,\eta(\Omega)}_{\text{Normalization Integral}}$$

- Maximize by choosing SDMEs such that the intensity fits the observed N events
- Accidental background subtracted in likelihood
- Normalization integral evaluated by a phase-space Monte Carlo sample with the acceptance $\eta(\Omega) = 0/1$

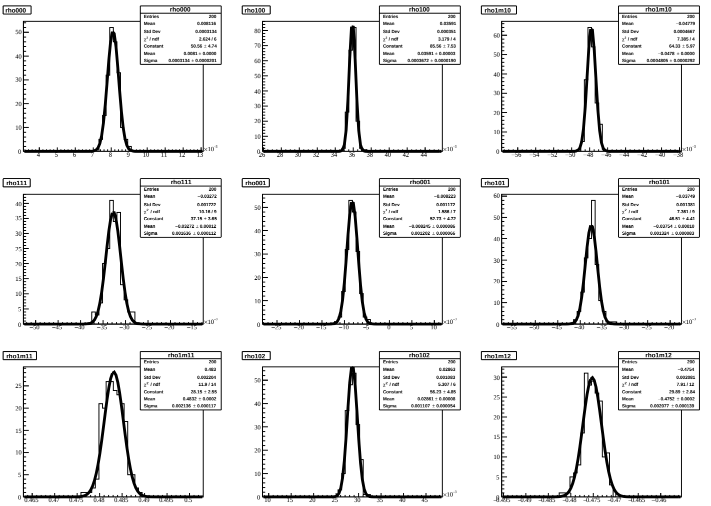
Result

$\gamma p \to \rho(770)p$



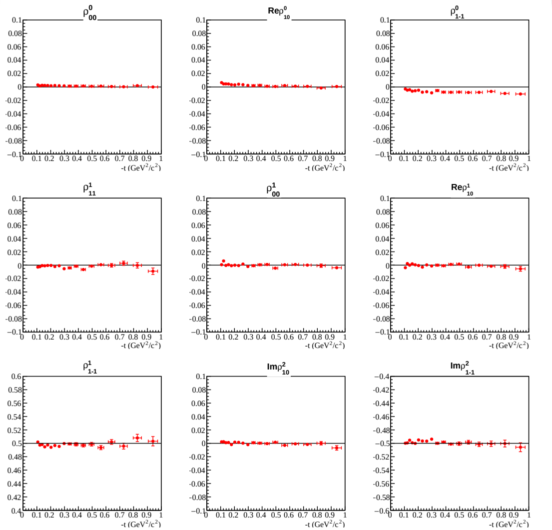
- Combined fit of 4 orientations with constraints
- Excellent agreement with JPAC for $t < 0.5\,\text{GeV}^2$
- Statistical uncertainties only
- Systematic studies presented today

Statistical Uncertainties: Bootstrapping



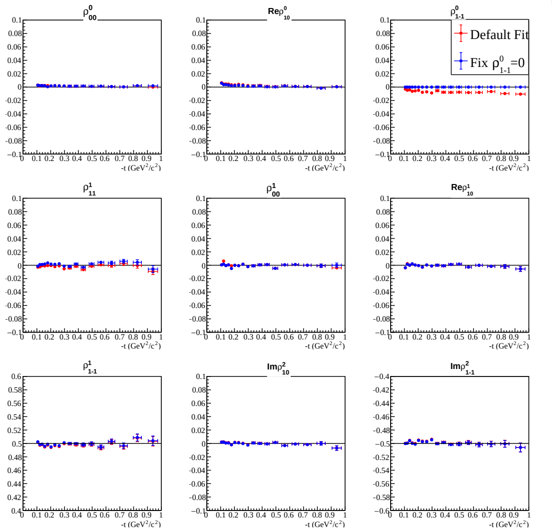
- Repeat fit 200 times by resampling the datasets
- Determine mean and variance via Gaussian fit, use for final result
- Gaussian variance ≈ 25% larger than Minuit uncertainty

Input/Output Test with Signal MC



- Significant effect for ρ^0_{1-1} for full t range
- Smaller effect on ρ^0_{10} near $t = 0.1$

Input/Output Test with Signal MC



- Significant effect for ρ^0_{1-1} for full t range
- Smaller effect on ρ^0_{10} near $t = 0.1$
- Nearly independent from other SDMEs
- Add deviation to systematic uncertainty

Barlow's Significance Test

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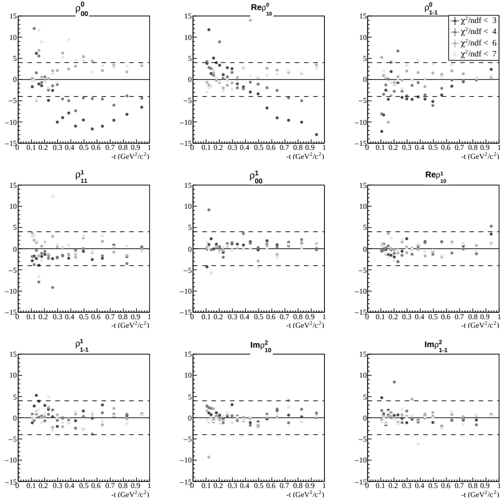
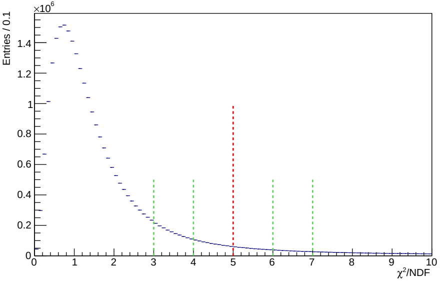
$$B = \frac{\Delta}{\sigma_B} = \frac{\rho - \rho_i}{\sqrt{|\sigma^2 - \sigma_i^2|}}$$

- Gauge statistical significance suggested by Barlow
- If $B < 1$: not significant
- If $B > 4$: take into account
- Else: discuss

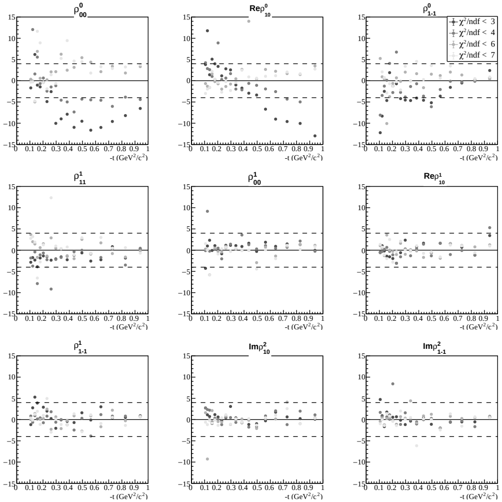
`arXiv:hep-ex/0207026`



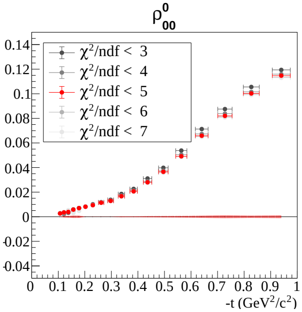
- Default: $\chi^2/\mathrm{ndf} < 5$
- Variation: ± 2 corresponds roughly to $\pm 10\%$ data
- Unpolarized ρ^0s clearly fail significance test



Kinematic Fit

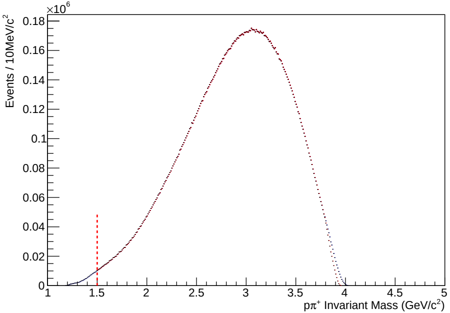


- Default: $\chi^2/\mathrm{ndf} < 5$
- Variation: ± 2 corresponds roughly to $\pm 10\%$ data
- Unpolarized ρ^0s clearly fail significance test
- Compute standard deviation from variations and use as systematic uncertainty

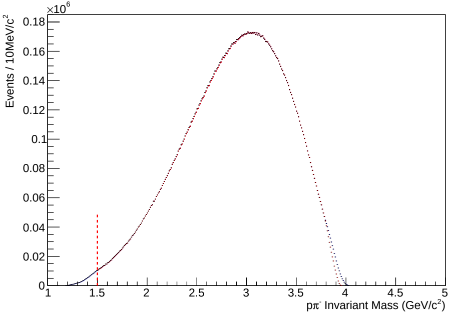


Effect of Target Excitation

$p\pi^+$ Invariant Mass

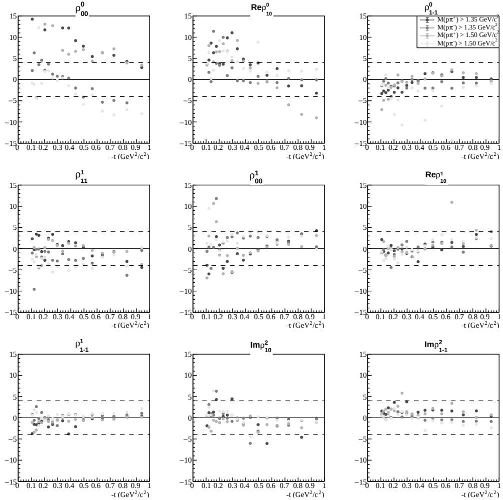


$p\pi^-$ Invariant Mass



- Nearly no evidence for baryon excitations after selection of $\rho(770)$ mass region
- Systematic study: cuts at $M(p\pi^{\pm}) > 1.35, 1.5\,\mathrm{GeV}/c^2$, data reduction maximal 0.6%

Effect of Target Excitation



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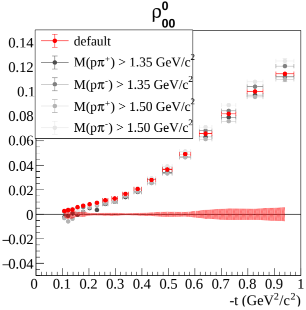
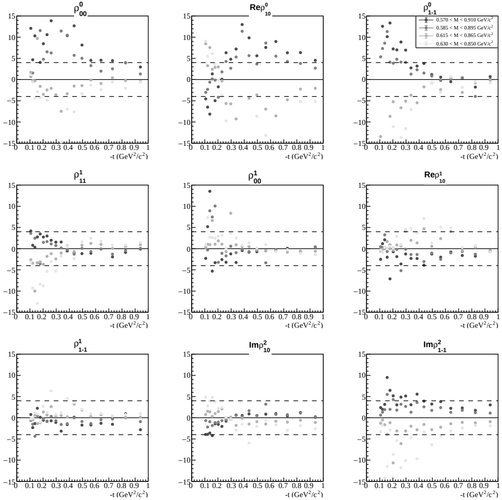
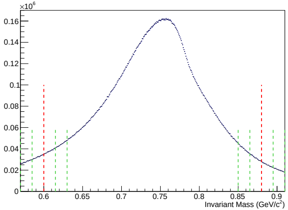


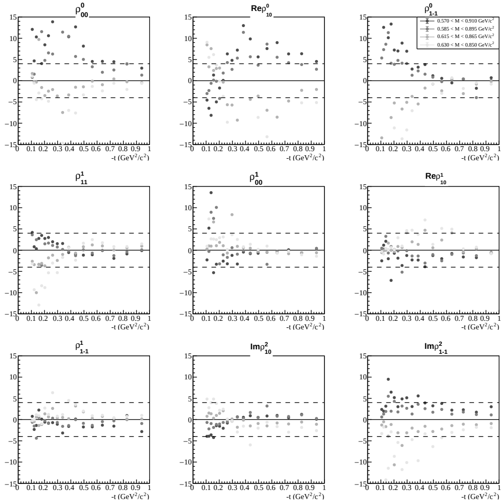
Invariant Mass



- Variation: $\pm 150, 300\mathrm{MeV}/c^2$ corresponds to maximum of $\pm 10\%$ data
- Several SDMEs fail significance test
- Compute standard deviation from variations and use as systematic uncertainty



Invariant Mass



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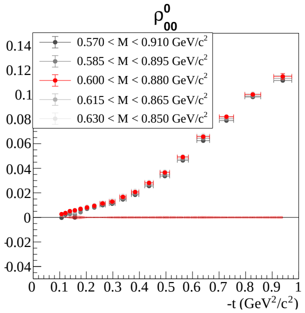
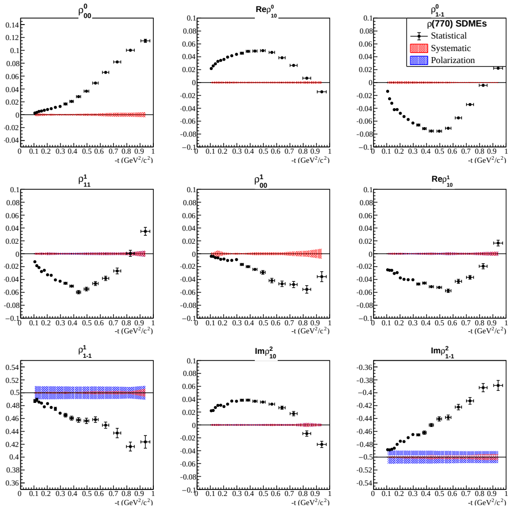
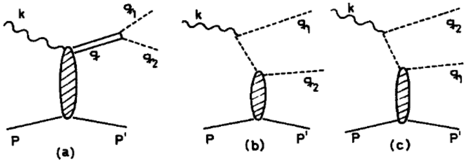


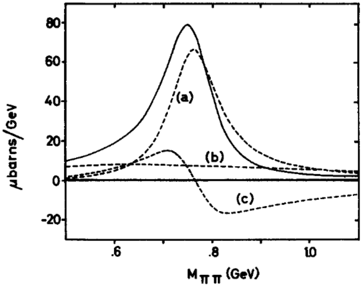
Beam Polarization and Summary



- Individual contributions added quadratically (plot has to be checked)
- 2.1% systematic uncertainty on P_{γ} added quadratically to $\rho^{1,2}$s only
- Uncertainty from Input/Output test not yet added
- No significant contribution from orientations of polarization

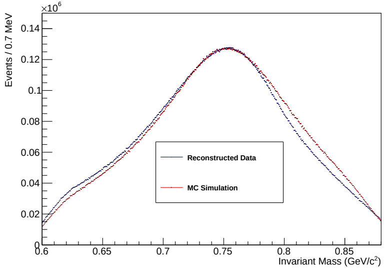
Excursion: Mass Dependence



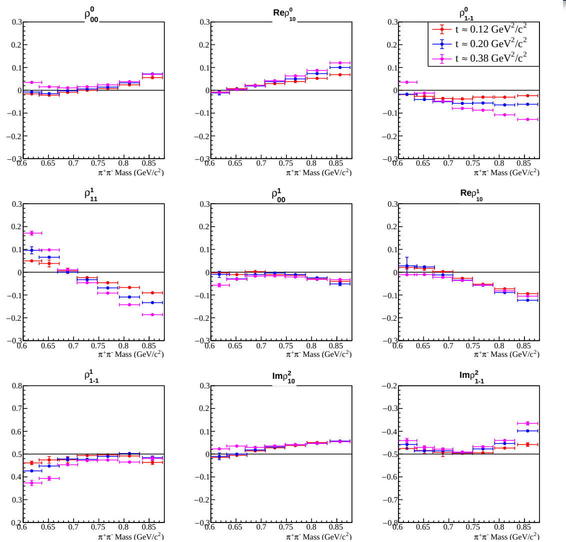


Soding, Phys. Lett. 19, 702 (1966)

- Breit-Wigner Mass observed to be 18 MeV/c^2 lower than PDG value
- Consistent with earlier observations
- Explained by interference with non-resonant processes (b) and (c)



Excursion: Mass Dependence



- Extract SDMEs as a function of $\pi^+\pi^-$ mass for each t bin
- Mass dependence increases with t as S-wave background increases
- ρ^1_{11} shows largest mass dependence, but effect seen in all
- Physics result instead of systematic error
- Narrower mass bin will increase statistical uncertainty

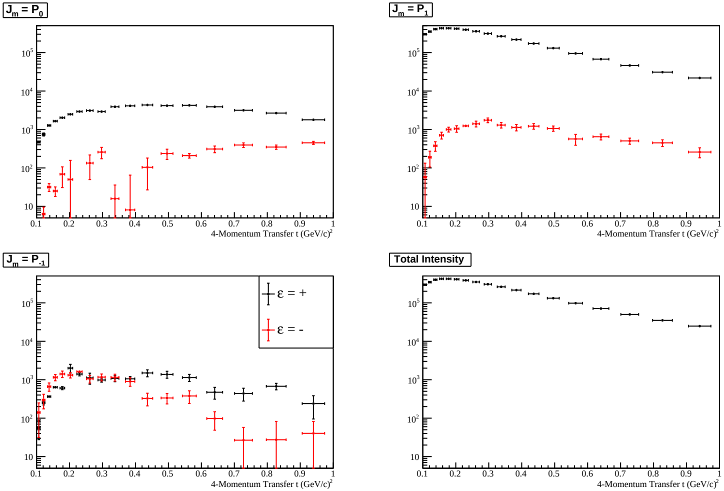
- Definition:

$$Z_\ell^m(\Omega,\Phi) \equiv Y_\ell^m(\Omega)e^{-i\Phi}.$$

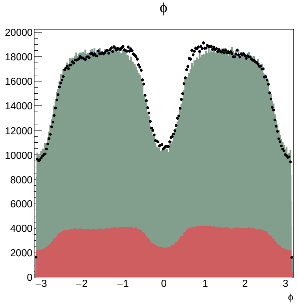
- Final formulation of intensity:

$$I(\Omega,\Phi) = 2\kappa\sum_k\left\{(1-P_\gamma)\left|\sum_{\ell,m}[\ell]^{(-)}_{m;k}\mathrm{Re}[Z_\ell^m(\Omega,\Phi)]\right|^2 + (1-P_\gamma)\left|\sum_{\ell,m}[\ell]^{(+)}_{m;k}\mathrm{Im}[Z_\ell^m(\Omega,\Phi)]\right|^2 + (1+P_\gamma)\left|\sum_{\ell,m}[\ell]^{(+)}_{m;k}\mathrm{Re}[Z_\ell^m(\Omega,\Phi)]\right|^2 + (1+P_\gamma)\left|\sum_{\ell,m}[\ell]^{(-)}_{m;k}\mathrm{Im}[Z_\ell^m(\Omega,\Phi)]\right|^2\right\}.$$

- Absorb factor $\sqrt{1 \pm P_\gamma}$ into amplitude
- Repeated $[l]^{\pm}_{m;k}$ have to be constrained
- Equivalent to decomposition into SDMEs



- Stable fit with all *P*-waves: $P^{\varepsilon=\pm}_{\ell=-1,0,1}$
- Small negative reflectivity contribution



Excursion: Polarized Reflectivity Amplitudes

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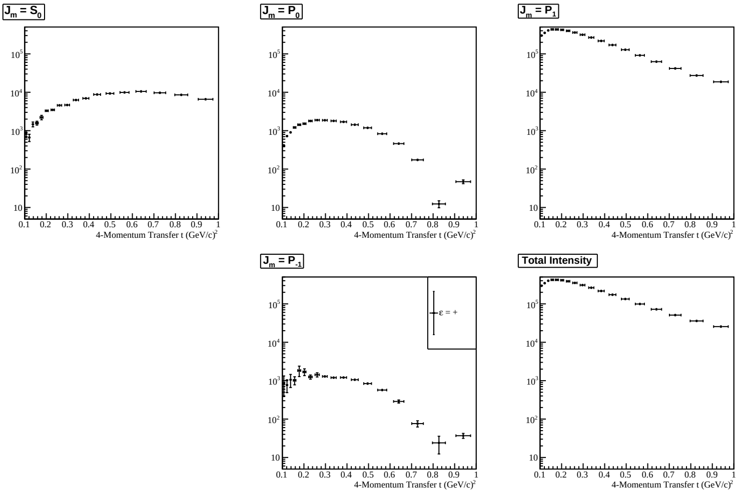
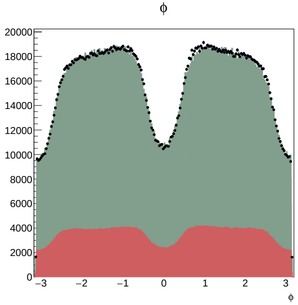


- Stable fit with all *P*-waves: $P^{\varepsilon=\pm}_{\ell=-1,0,1}$
- Small negative reflectivity contribution neglected
- Interference with *S*-wave explains φ asymmetry



Summary

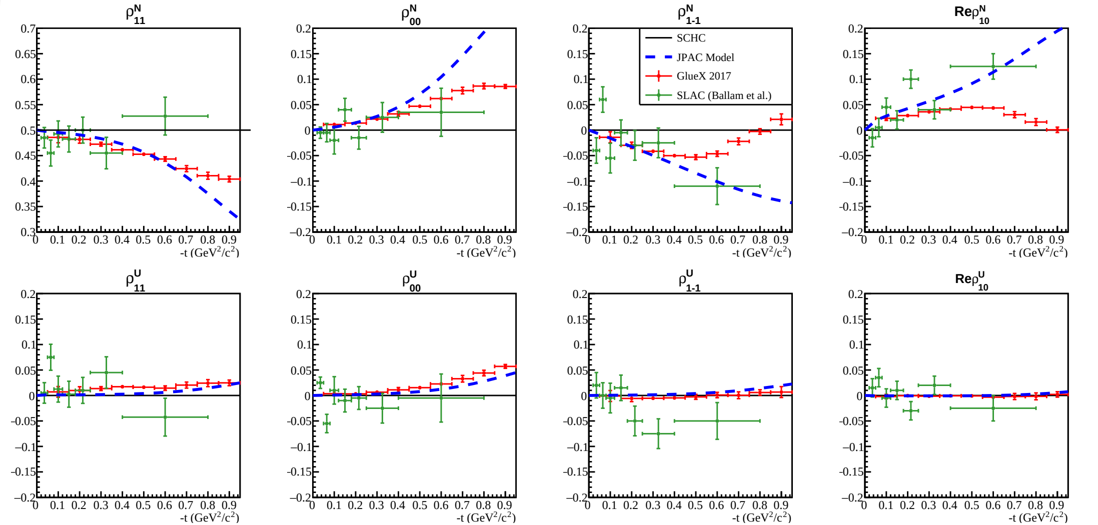
Systematic Studies

- Analysis note revised and updated
- Presented proposal for systematic uncertainties
- Take out mass dependence from systematics, since it is physics?
- Additional sources?

Physics Results for Paper

- Main result are SDMEs as a function of t, comparison with JPAC
- Linear combinations for natural/unnatural parity exchange contribution
- Relationship and constraints between individual SDMEs
- Mass dependence of SDMEs and interference with S-wave
- Connection to polarized reflectivity amplitudes

(Un)Natural Parity Exchange



SDMEs Expressed in Amplitudes

V. Mathieu, [Phys.Rev.D 100 (2019) 5, 054017]

$$\rho^{\alpha,\ell\ell'}_{mm'} = {}^{(+)}\rho^{\alpha,\ell\ell'}_{mm'} + {}^{(-)}\rho^{\alpha,\ell\ell'}_{mm'} \, .$$

$$\begin{aligned} {}^{(\epsilon)}\rho^{0,\ell\ell'}_{mm'} &= \kappa \sum_k \left([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} \right. \\ &\qquad \left. + (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \right) , \end{aligned} \tag{D8a}$$

$$\begin{aligned} {}^{(\epsilon)}\rho^{1,\ell\ell'}_{mm'} &= -\epsilon\kappa \sum_k \left((-1)^m [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} \right. \\ &\qquad \left. + (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \right) , \end{aligned} \tag{D8b}$$

$$\begin{aligned} {}^{(\epsilon)}\rho^{2,\ell\ell'}_{mm'} &= -i\epsilon\kappa \sum_k \left((-1)^m [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} \right. \\ &\qquad \left. - (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \right) , \end{aligned} \tag{D8c}$$