PWA Challenge with polarized photon beam

Florida International University 2020

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Polarized moments

$$I(\Omega,\Phi) = I^0(\Omega) - P_\gamma I^1(\Omega)\cos 2\Phi - P_\gamma I^2(\Omega)\sin 2\Phi$$

The angular distribution can be expanded in unpolarized moment H^0 and polarized moments ***H***=(H^1 , H^2 , H^3)

$$I^0(\Omega) = \sum_{L,M\geqslant 0}\left(\frac{2L+1}{4\pi}\right)\tau(M)H^0(LM)d^L_{M0}(\theta)\cos M\phi$$

$$I^1(\Omega) = -\sum_{L,M\geqslant 0}\left(\frac{2L+1}{4\pi}\right)\tau(M)H^1(LM)d^L_{M0}(\theta)\cos M\phi,$$

$$I^2(\Omega) = 2\sum_{L,M>0}\left(\frac{2L+1}{4\pi}\right)\mathrm{Im}\,H^2(LM)d^L_{M0}(\theta)\sin M\phi$$

$$\tau(M) = (2-\delta_{M,0})$$

Φ - angle between γ polarization vector $\vec{\epsilon}'$ and production plane
Ω- direction of η in helicity frame
P_γ is the degree of linear polarization
$A_{\lambda;\lambda_1\lambda_2}(\Omega)$-the reaction amplitude

Using the following wave set : $[\ell]^{(\epsilon)}_{m;k} = \left\{S_0^{(+)}, P_{0,1}^{(+)}, D_{0,1,2}^{(+)}\right\}_{k=0}$

From an experimental perspective, the moments are extracted from the angular distribution without assuming a particular wave content

one can extract the moments up to L = 4. In addition, since there are only waves with positive m components the moments full the following relation $\mathrm{Im}\,H^2(LM) = -H^1(LM)$ for $M\geqslant 1$. $0\leqslant L\leqslant 4$ and $0\leqslant M\leqslant L$

Polarized moments calculated with partial waves

The moments are expressed in terms of the $\eta'\pi^0$ SDME:

$$H^0(LM) = \sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M} \, \rho^{\alpha,\ell\ell'}_{mm'}$$

$$\boldsymbol{H}(LM) = -\sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M} \, \boldsymbol{\rho}^{\ell\ell'}_{mm'}$$

$$
\begin{aligned}
{}^{(\epsilon)}\rho^{0,\ell\ell'}_{mm'} &= \kappa \sum_k \Big([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\quad + (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) , \\
{}^{(\epsilon)}\rho^{1,\ell\ell'}_{mm'} &= -\epsilon\kappa \sum_k \Big((-1)^{m} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\quad + (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) , \\
{}^{(\epsilon)}\rho^{2,\ell\ell'}_{mm'} &= -i\epsilon\kappa \sum_k \Big((-1)^{m} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\quad - (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) , \\
{}^{(\epsilon)}\rho^{3,\ell\ell'}_{mm'} &= \kappa \sum_k \Big([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\quad - (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) .
\end{aligned}
$$

Polarized moments calculated with partial waves

$$H^0(00) = H^1(00) + 2\left[|P_1^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2\right] , \quad \text{(E1a)}$$

$$H^1(00) = 2\left[|S_0^{(+)}|^2 + |P_0^{(+)}|^2 + |D_0^{(+)}|^2\right] , \quad \text{(E1b)}$$

$$H^0(10) = H^1(10) + \frac{4}{\sqrt{5}}\mathrm{Re}(P_1^{(+)} D_1^{(+)*}) , \quad \text{(E1c)}$$

$$H^1(10) = \frac{8}{\sqrt{15}}\mathrm{Re}(P_0^{(+)} D_0^{(+)*}) + \frac{4}{\sqrt{3}}\mathrm{Re}(S_0^{(+)} P_0^{(+)*}) , \quad \text{(E1d)}$$

$$H^0(11) = H^1(11) + 2\sqrt{\frac{2}{5}}\mathrm{Re}(P_1^{(+)} D_2^{(+)*}) , \quad \text{(E1e)}$$

$$H^1(11) = \frac{2}{\sqrt{5}}\mathrm{Re}(P_0^{(+)} D_1^{(+)*}) - \frac{2}{\sqrt{15}}\mathrm{Re}(P_1^{(+)} D_0^{(+)*}) + \frac{2}{\sqrt{3}}\mathrm{Re}(S_0^{(+)} P_1^{(+)*}) , \quad \text{(E1f)}$$

$$H^0(20) = H^1(20) - \frac{2}{5}|P_1^{(+)}|^2 + \frac{2}{7}|D_1^{(+)}|^2 - \frac{4}{7}|D_2^{(+)}|^2 , \quad \text{(E1g)}$$

$$H^1(20) = \frac{4}{5}|P_0^{(+)}|^2 + \frac{4}{7}|D_0^{(+)}|^2 + \frac{4}{\sqrt{5}}\mathrm{Re}(S_0^{(+)} D_0^{(+)*}) , \quad \text{(E1h)}$$

$$H^0(21) = H^1(21) + \frac{2}{7}\sqrt{6}\,\mathrm{Re}(D_1^{(+)} D_2^{(+)*}) , \quad \text{(E1i)}$$

$$H^1(21) = \frac{2}{\sqrt{5}}\mathrm{Re}(S_0^{(+)} D_1^{(+)*}) + \frac{2\sqrt{3}}{5}\mathrm{Re}(P_0^{(+)} P_1^{(+)*}) + \frac{2}{7}\mathrm{Re}(D_0^{(+)} D_1^{(+)*}) , \quad \text{(E1j)}$$

$$H^0(22) = \frac{2}{\sqrt{5}}\mathrm{Re}(S_0^{(+)} D_2^{(+)*}) - \frac{4}{7}\mathrm{Re}(D_0^{(+)} D_2^{(+)*}) ,$$

Non-zero P-wave would be directly observable from its interference with even waves in moments with odd angular momenta.

Exotic wave in terms of moments

$$|P_1^{(+)}|^2 = \frac{1}{2}\Delta(00) + \frac{21}{8}\Delta(40) + \frac{3}{4}\sqrt{\frac{35}{2}}\Delta(44)$$
$$= -\frac{5}{\sqrt{6}}\Delta(22) + \frac{15}{8}\Delta(40) + \frac{3}{4}\sqrt{\frac{5}{14}}\Delta(44)$$
$$= -\frac{5}{2}\Delta(20) - \frac{15}{8}\Delta(40) + \frac{3}{4}\sqrt{\frac{35}{2}}\Delta(44)$$
$$= -\frac{5}{18}\Delta(00) - \frac{35}{36}\Delta(20) - \frac{35}{6\sqrt{6}}\Delta(22) .$$

$$\Delta(LM) = H^0(LM) - H^1(LM)$$

$$H^1(22) = H^0(22) + \frac{\sqrt{6}}{7}|D_1^{(+)}|^2 + \frac{\sqrt{6}}{5}|P_1^{(+)}|^2, \quad \text{(E1l)}$$

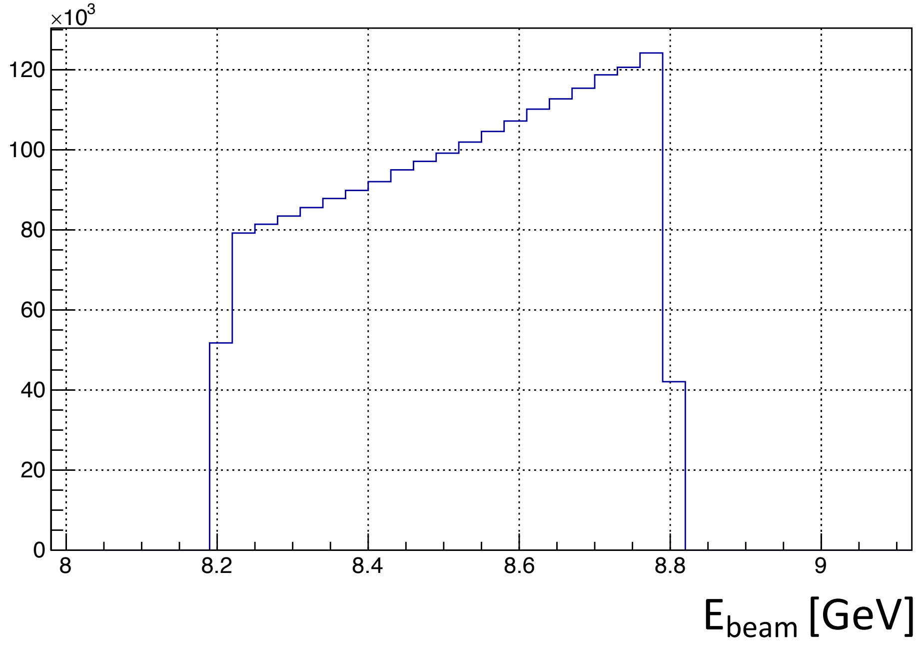
Generated 2*10⁶ $(p\eta'\pi^0)$events with AmpTools

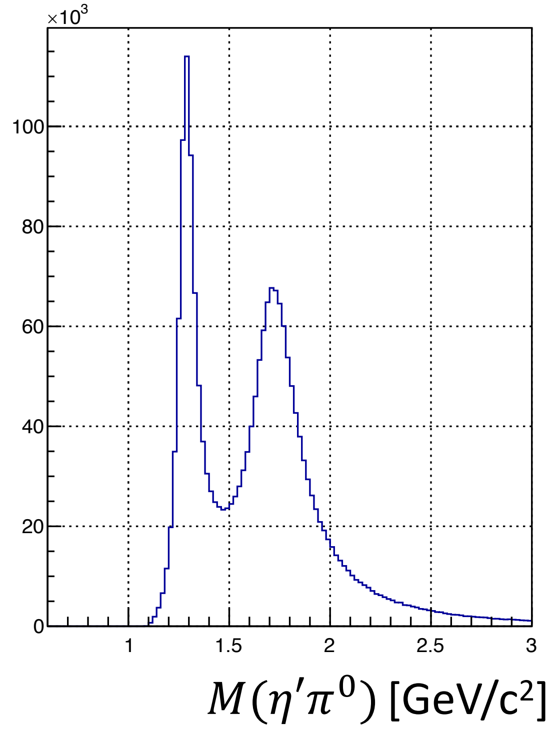
Generated amplitudes are
- P1/π_1(1600 MeV) (exotic)
- D1/a2 (1320 MeV)
- D1/a2' (1700 MeV)

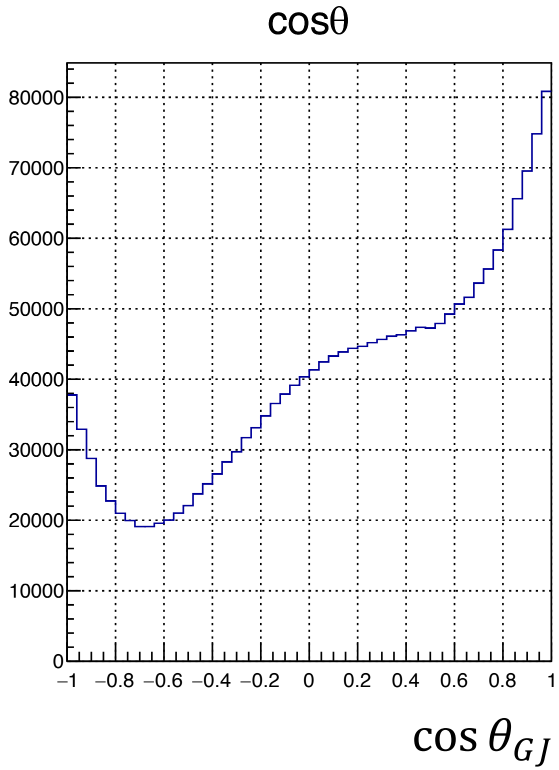
Φ=1.77 Deg.

P_γ =0.3

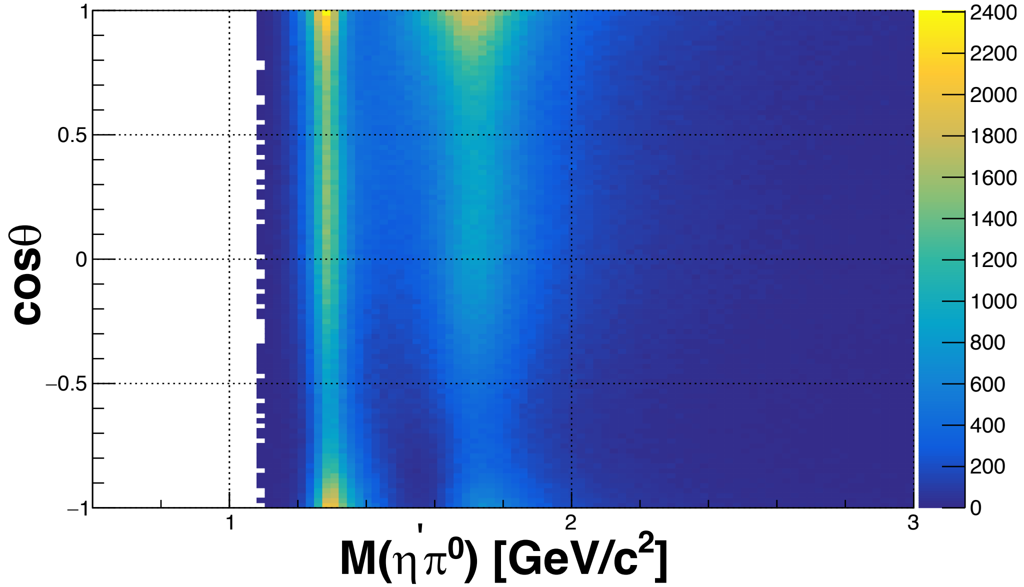
J	M	ε	Real	Imaginary	BW Mass	BW Width
1	0, 1	+1	50	50	1.564	0.492
2	0,1,2	+1	150	150	1.306	0.114
2	0,1,2	+1	200	200	1.722	0.247

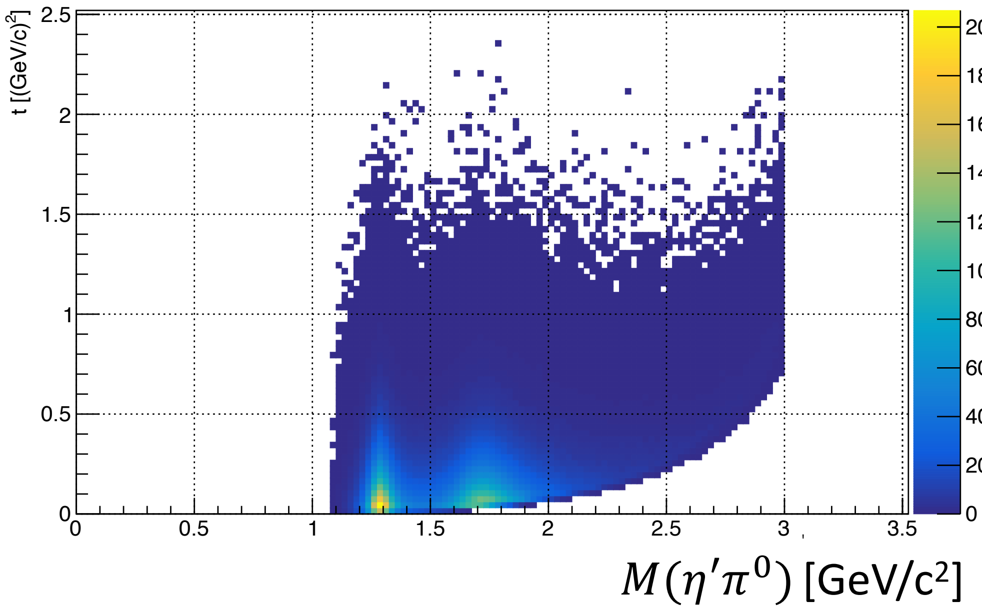


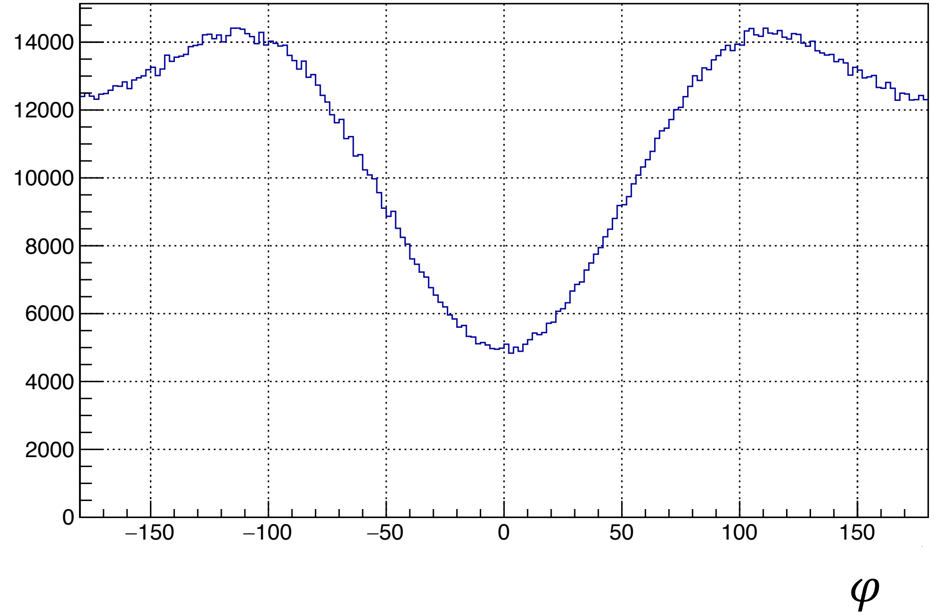


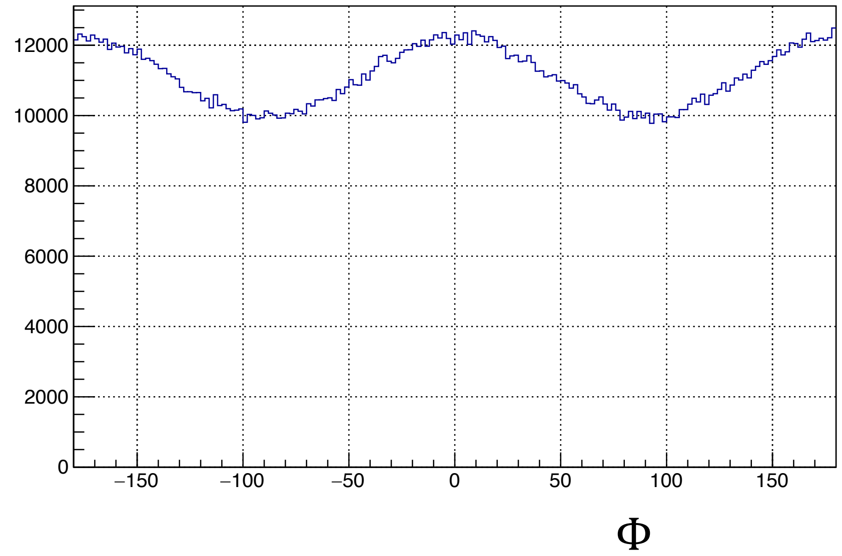


Generated $2*10^6$ $(p\eta'\pi^0)$events with AmpTools









Procedure

1. Fit intensity to find partial waves.
2. Calculate moments in terms of partial waves
3. Compare results from two different fits
   - Fit 1 : fit with true wave set
   - Fit 2: fit with true wave set plus S wave
4. Plot moments by weighting each event by corresponding intensity as defined in Mathiew et al.

$$H^0(LM) = \frac{P_\gamma}{2} \int_\circ I(\Omega, \Phi)\, d^L_{M0}(\theta) \cos M\phi,$$

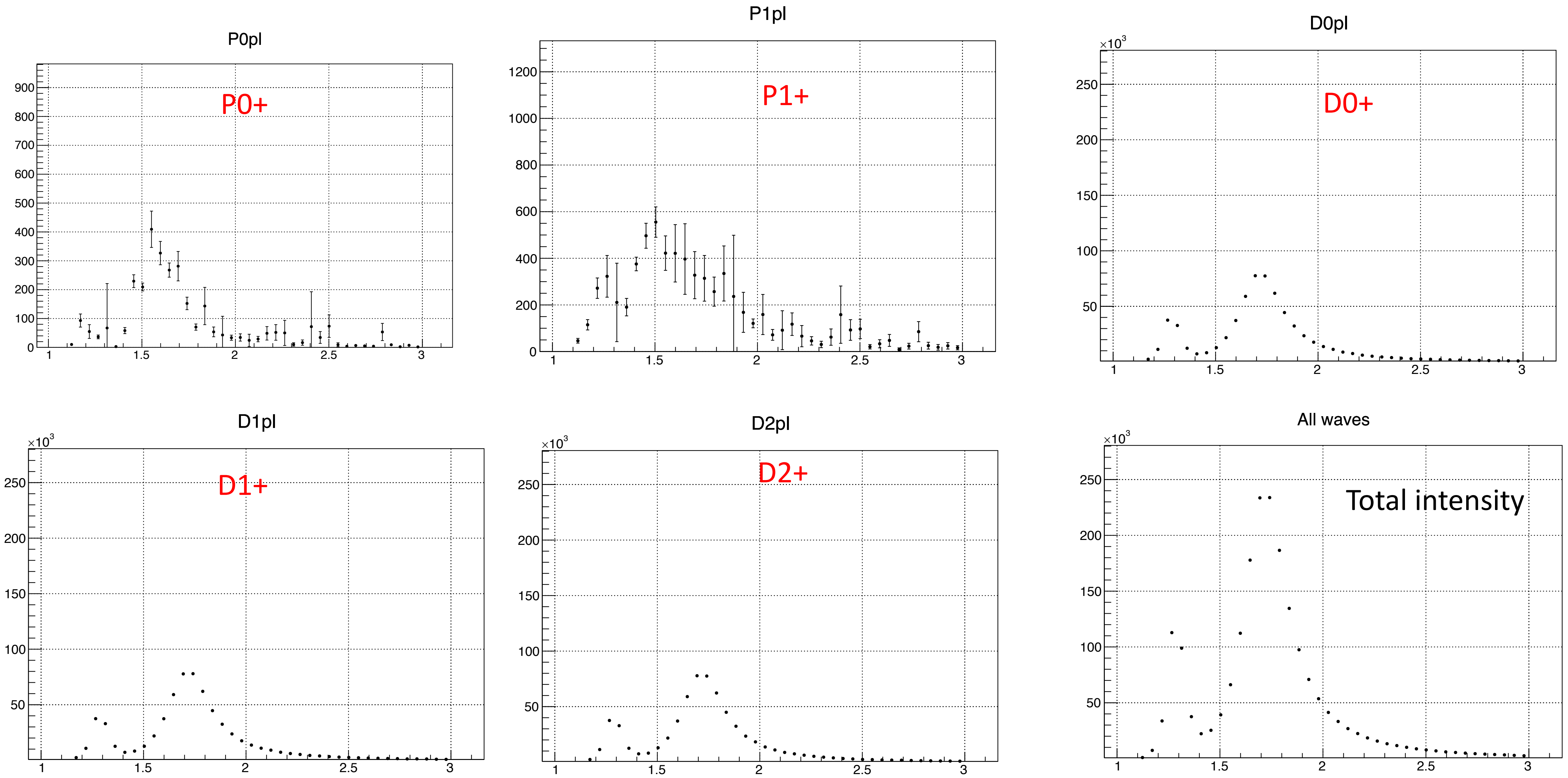
$$H^1(LM) = \int_\circ I(\Omega, \Phi)\, d^L_{M0}(\theta) \cos M\phi \, \cos 2\Phi,$$

$$\operatorname{Im} H^2(LM) = -\int_\circ I(\Omega, \Phi)\, d^L_{M0}(\theta) \sin M\phi \, \sin 2\Phi,$$

with $\int_\circ = (1/\pi P_\gamma) \int_0^\pi \sin\theta \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{2\pi} \mathrm{d}\Phi$

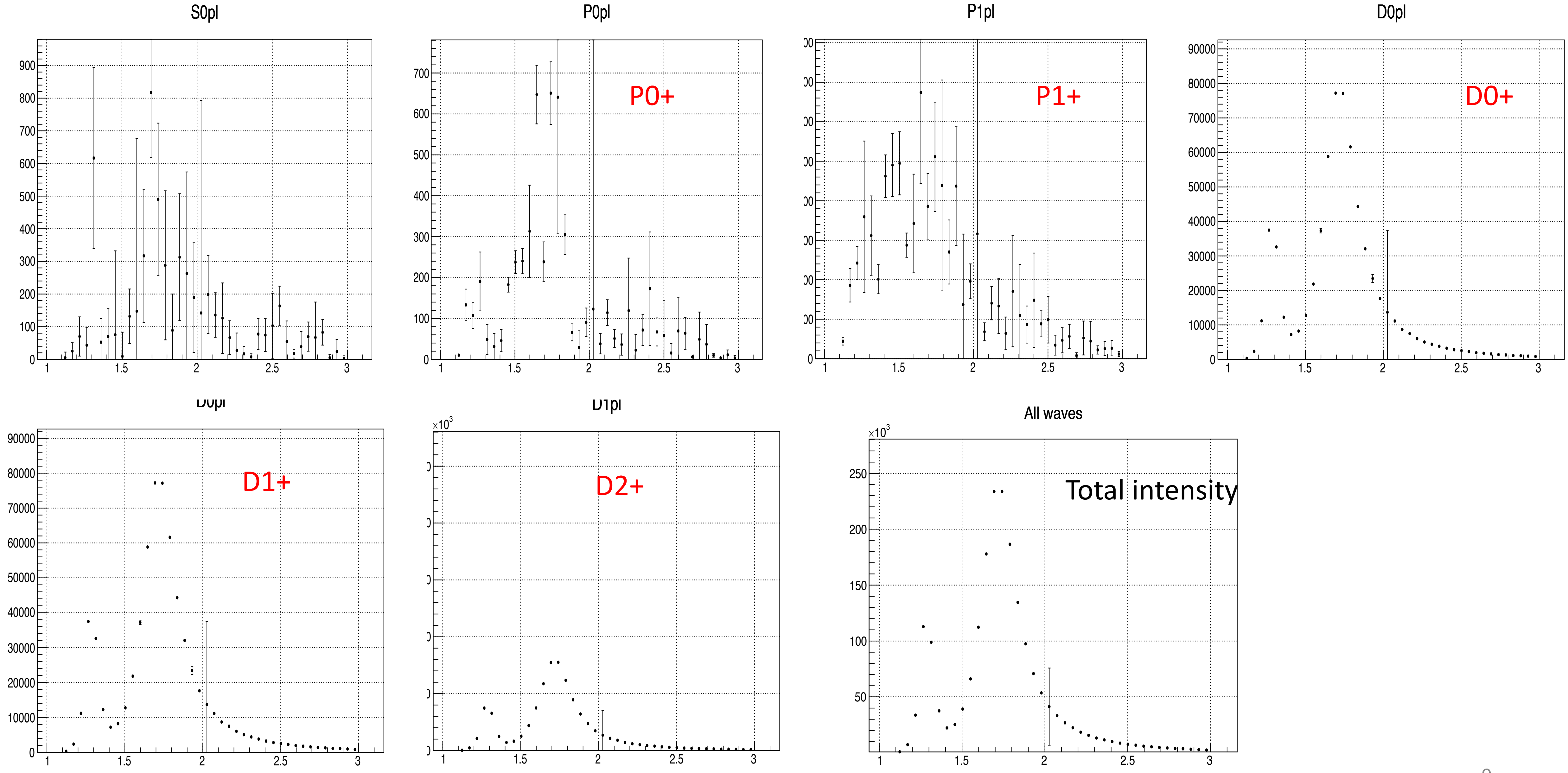
Fit 1 results (fitting in M and t bins)

Amplitudes used in fitting are P0+, P1+, D0+, D1+, D2+



Fit 2 results (fitting in M and t bins)

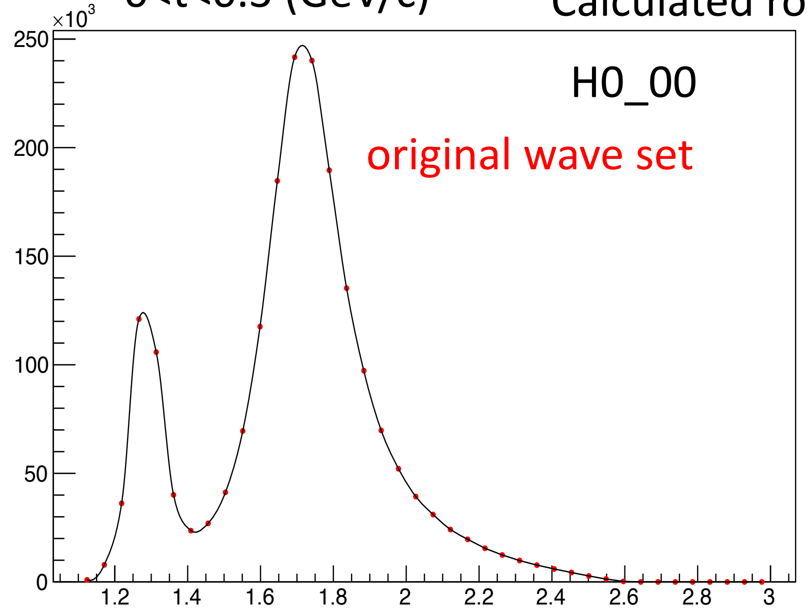
Amplitudes used in fitting are S0-, P0+, P1+, D0+, D1+, D2+

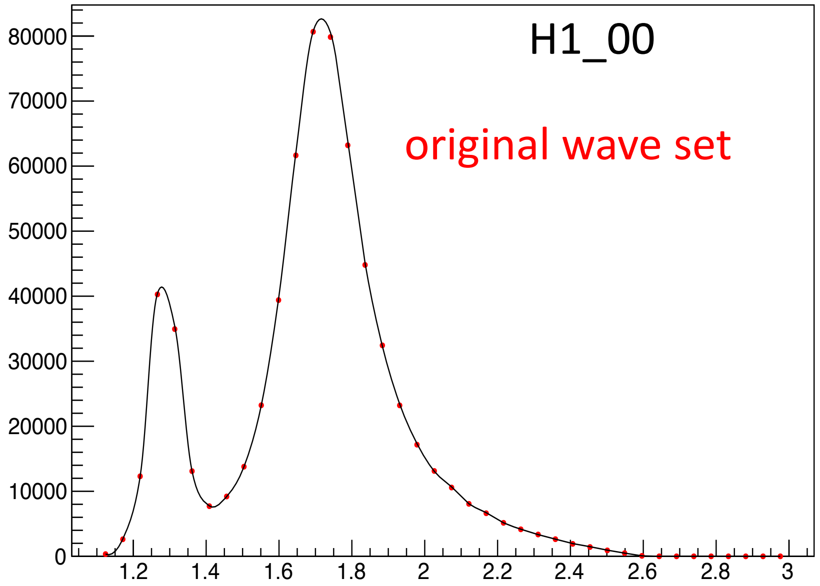


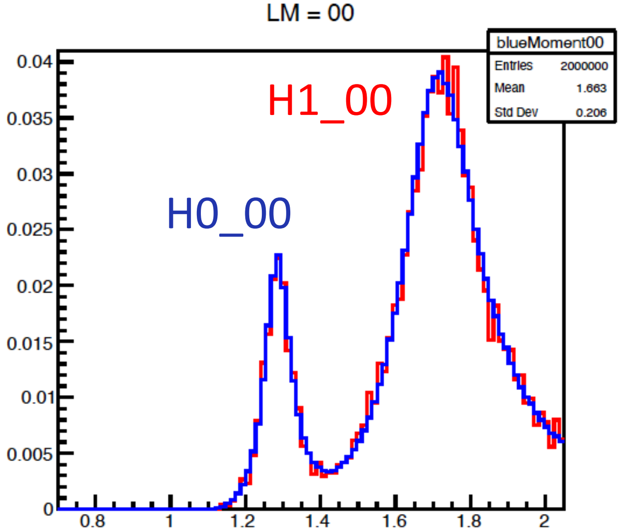
Polarized moments calculated with partial waves

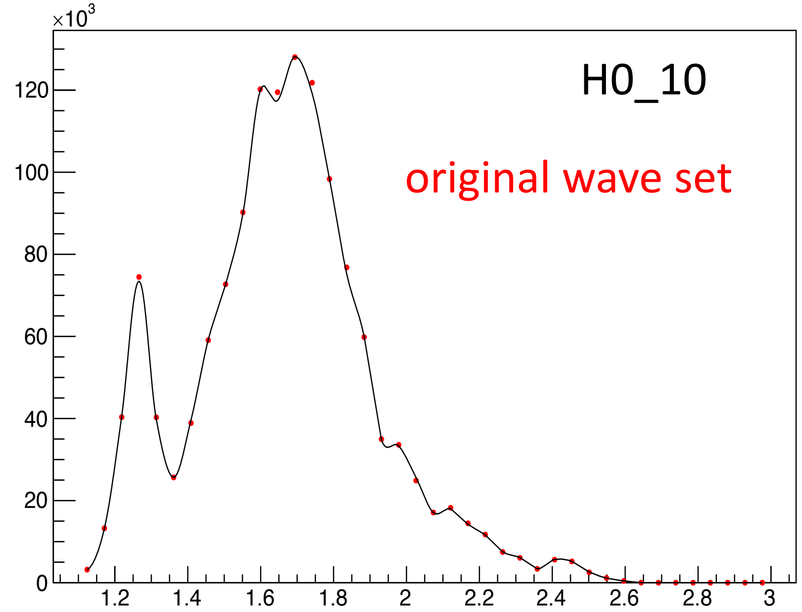
0<t<0.3 (GeV/c)^2 Calculated rom fitted amplitudes

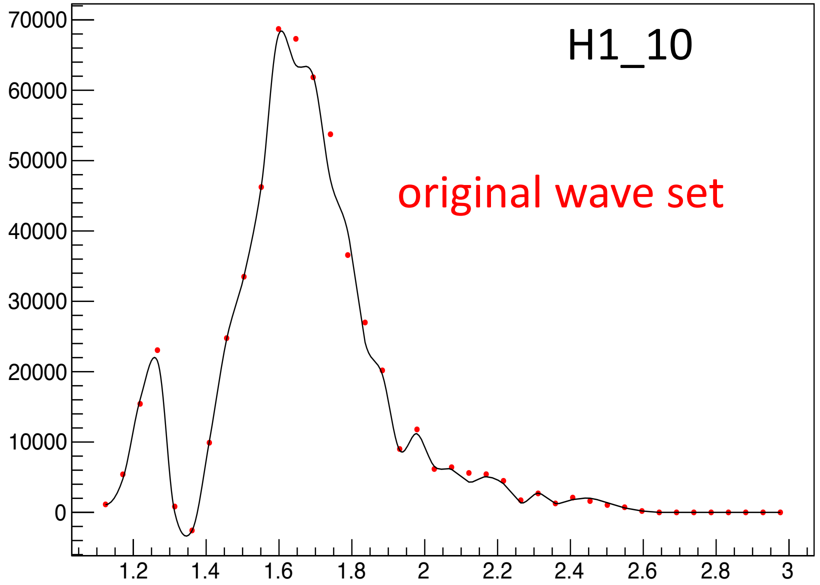
Obtained by weighting events of data sample

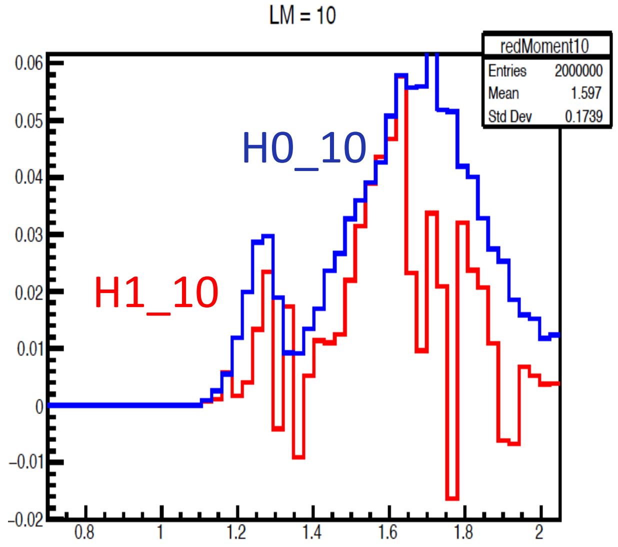










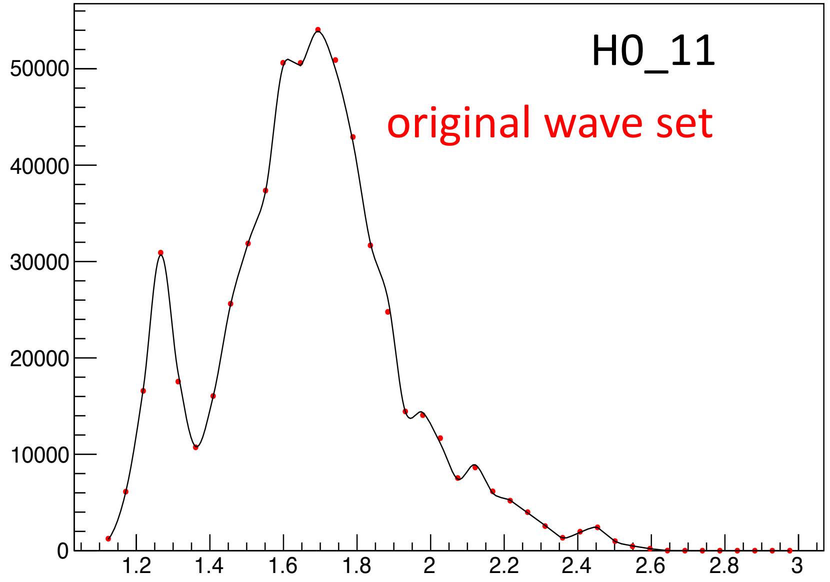


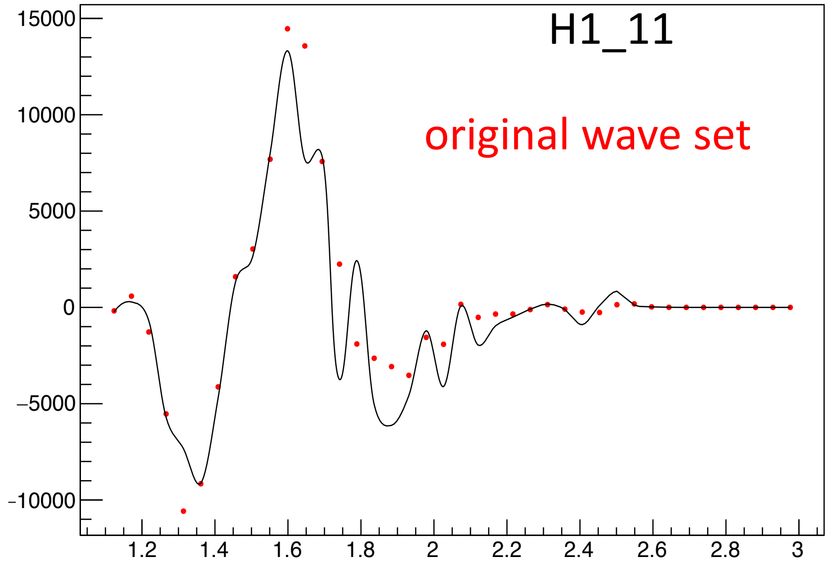
Polarized moments calculated with partial waves

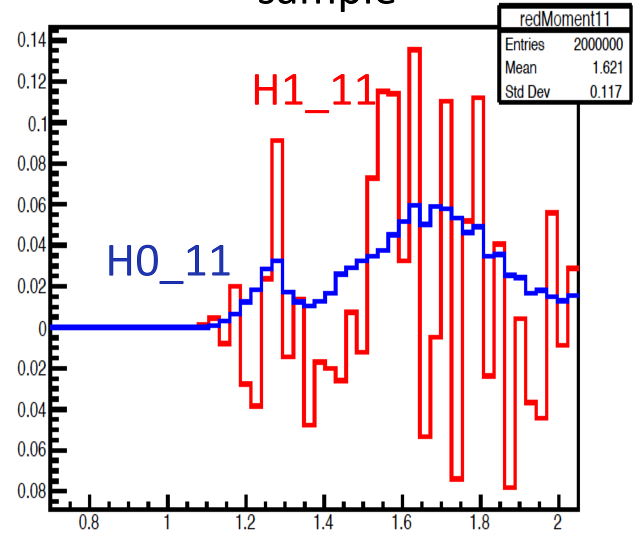
0<t<0.3 (GeV/c)[2]

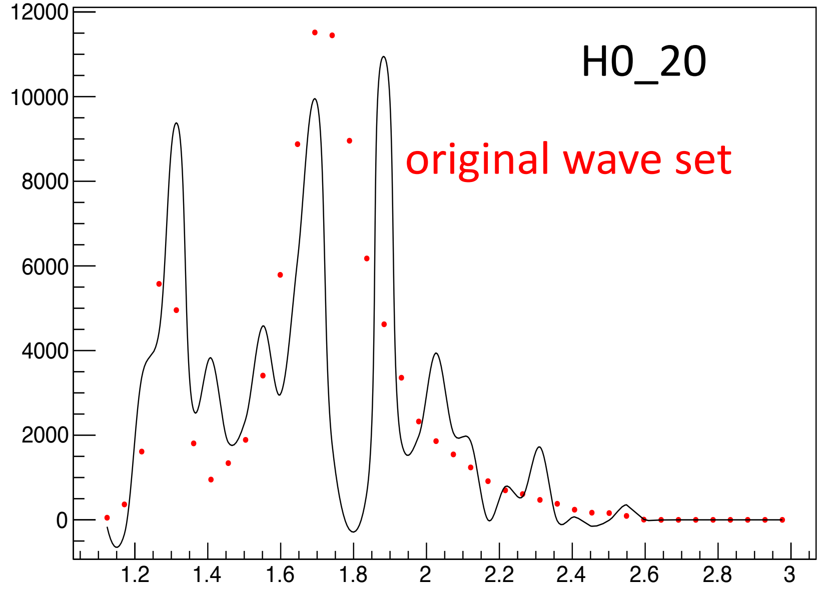
Calculated rom fitted amplitudes

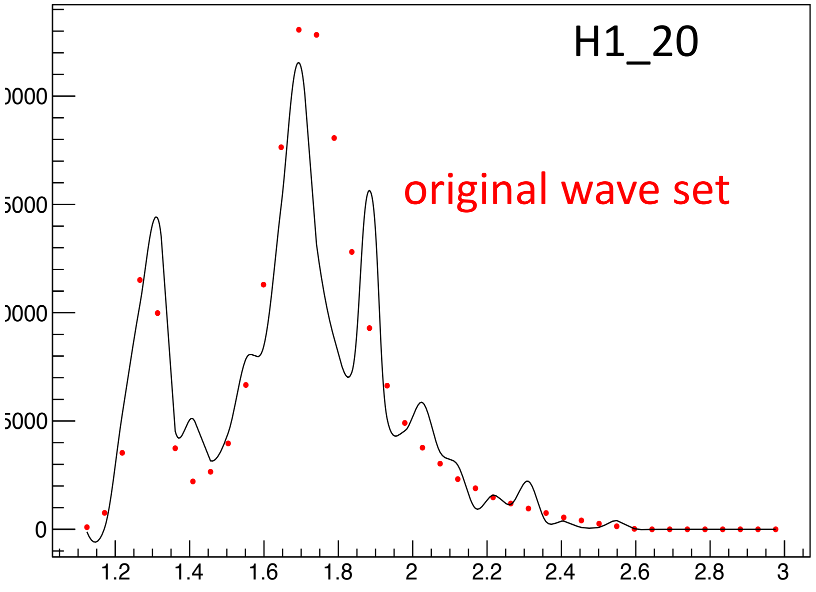
Obtained by weighting events of data sample

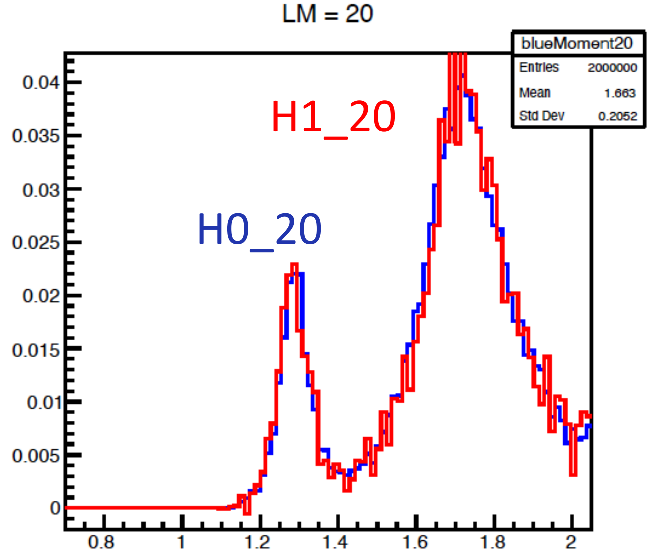












Polarized moments calculated with partial waves

$0<t<0.3\ (GeV/c)^2$

Calculated rom fitted amplitudes

Obtained by weighting events of data sample