

Track Find and Fitting

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- Track finding
- Wire-based fitting
- Time-based fitting

Tracking Sequence in words

• Step 1: Track finding

- Helical model – assume B-field is constant
- Look for clusters of hits close in position in adjacent (or close by) layers to form segments
 - No drift time information used at this time
- Link segments together to form “track candidates”
- Fit with Riemann Helical Fit to obtain estimates for tracking parameters (momentum and position at the vertex)

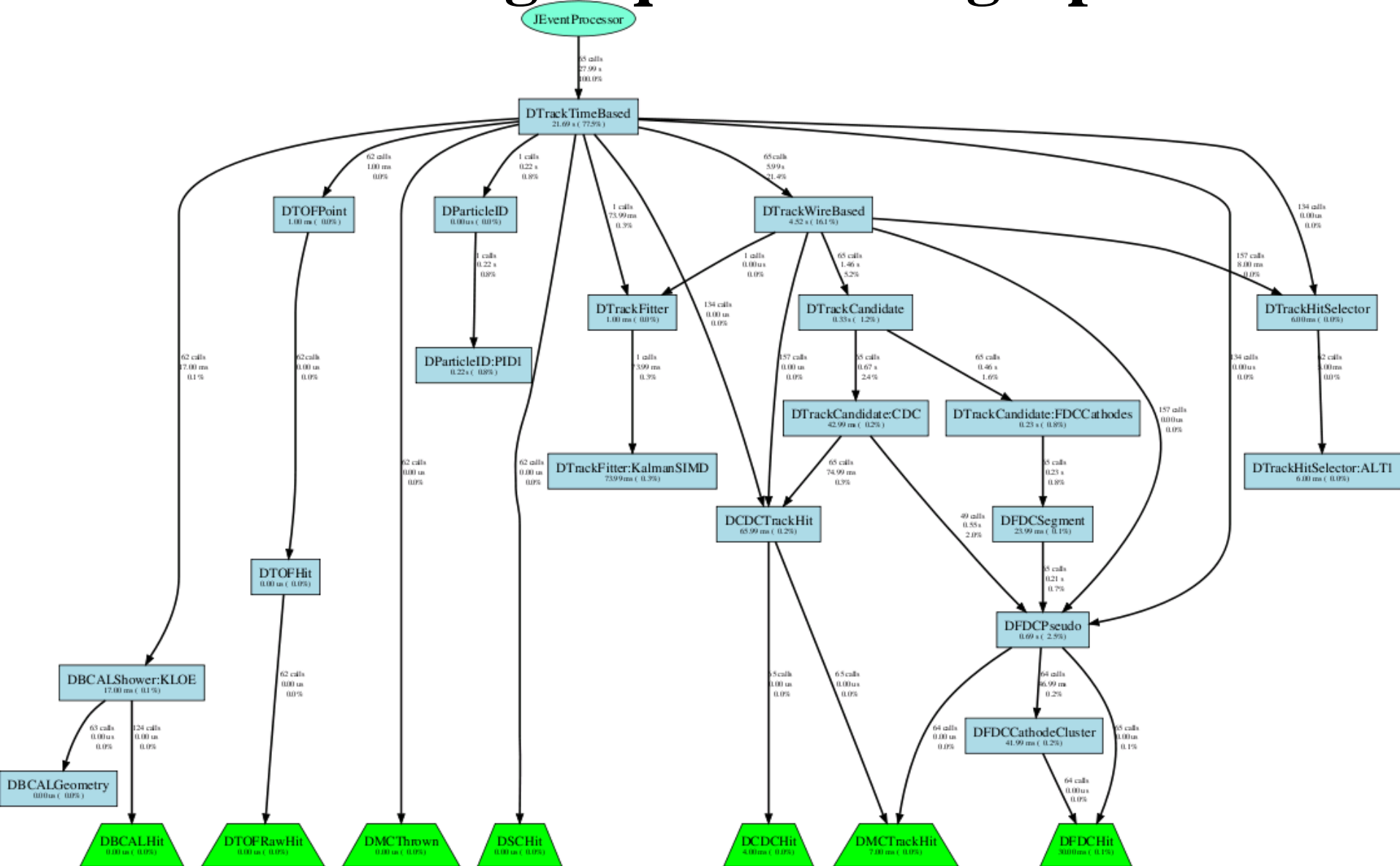
• Step 2: Wire-based fitting

- Fit the track candidates using the full knowledge of the non-uniform magnetic field but still without drift time data
- Fit multiple times with different mass hypotheses
 - {Proton, π^+ } for positive particles, π^- for negative particles
 - Take into account energy loss (important for low-momentum particles)

• Step 3: Time-based fitting

- Refit wire-based tracks using drift time information

Tracking sequence call graph



Track finding in CDC

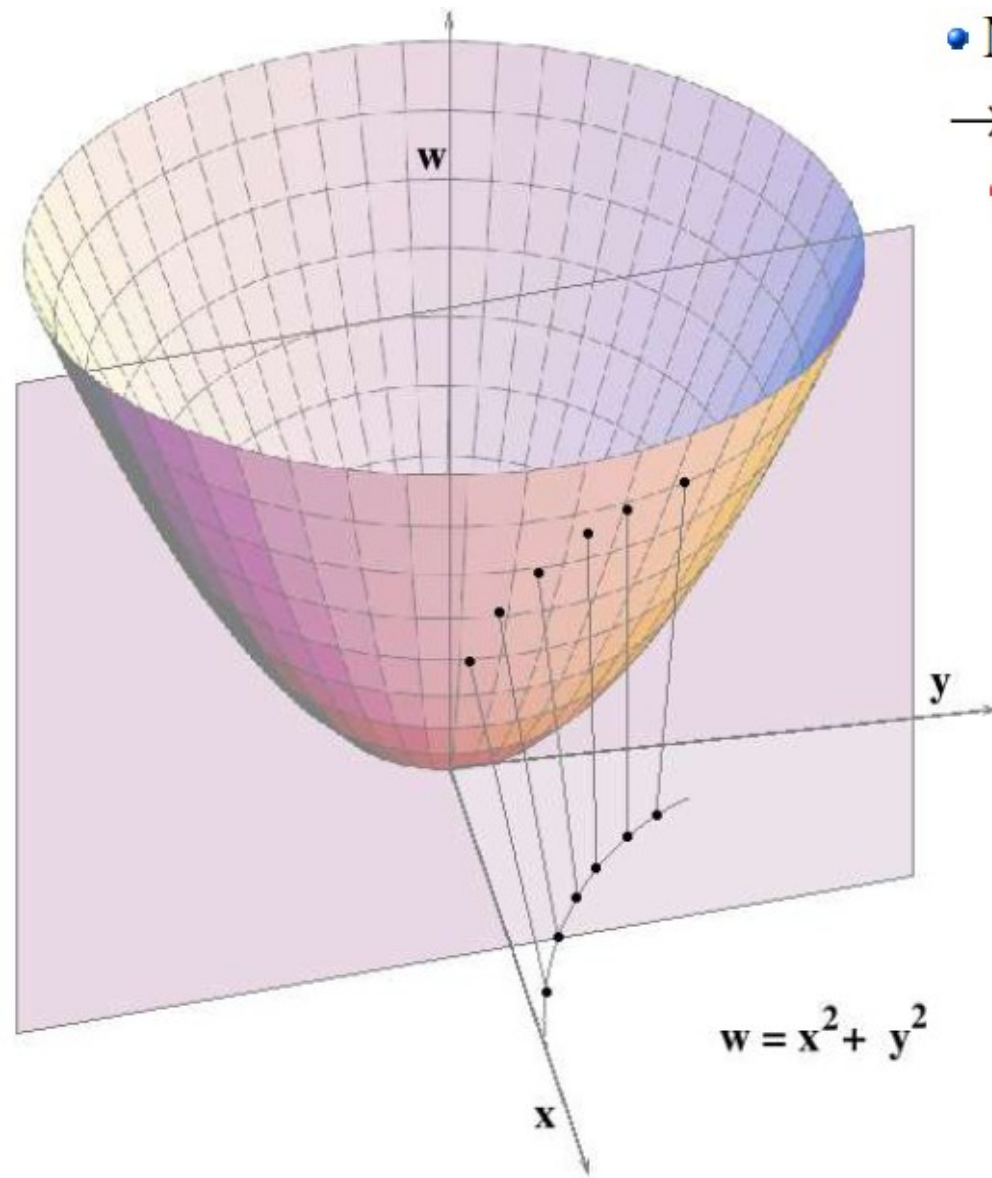
- Link hits together in each axial layer to form “seeds”
- Link seeds between axial layers together
- Find intersections between axial layers and stereo layers
- Use helical model to form track candidates → *DTrackCandidate: CDC*

Track finding in FDC

- Associate adjacent cathode hits into clusters → *DFDCCathodeClusters*
- Find centroids in cathode planes and match strips with wires to form space points → *DFDCPseudo*
- Link space points in adjacent layers in each package into track segments → *DFDCSegment*
- Link segments by swimming through the field from package to package to form track candidates → *DTrackCandidate:FDCCathodes*

Obtain preliminary momentum and position parameters for input to wire-based fit using **Riemann Helical Fit**

Riemann Helical Fit

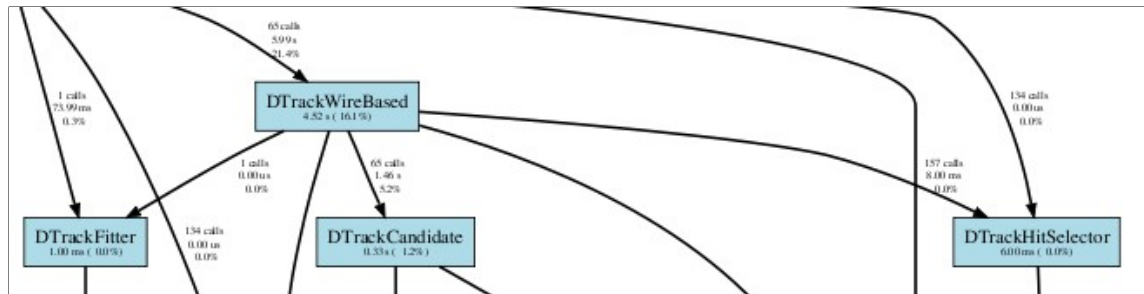


- Map points on circle in xy -projection
→ points on circular paraboloid surface
- Finding center, radius of circle → fitting plane in xyw -space
- Extension to helical path:
 - Compute arc lengths s from point to point in xyz -space
 - Linear regression of s on z → tangent of dip angle $\tan \lambda$

Class= *DHelicalFit*

R. Fruhwirth, et al., NIM **A490**(2002)366-378.

Step 2: Wire-based track fitting



- Starting with parameters from *DTrackCandidate*, swim out to select hits (using *DTrackHitSelector*) belonging to the track to be used in the fit
- Fit (with the algorithm chosen for *DTrackFitter*) using the wire positions for coordinates of the CDC hits and the position along the wire computed from the cathodes for the FDC → *DTrackWireBased*

Track Fitting Algorithm

DTrackFitter:KalmanSIMD_ALT1

- **Kalman Filter** is engine of Wire-based and Time-based fitting
- Track described by **5-parameter state vector** at each point along its path
- State vector propagated step-by-step toward target
 - Account for **multiple scattering** and **energy loss** at each step
 - Does not require inversions of large matrices (at most 2x2 for FDC hits)
 - Jacobian matrix elements computed analytically
- State vector treated as a perturbation to an initial guess
 - Generate **reference trajectory** by swimming with initial parameters from the target
- Iterate up to 20 times, checking for χ^2 convergence
 - Regenerate reference trajectory based on results of previous pass

Kalman Filter Algorithm

Start with guess from track finding or wire-based stages

Seed S^0

Propagate state S and covariance C through magnetic field

$$\begin{aligned} S_k^{k-1} &= S_k^{k-1,0} + F_k (S_{k-1}^{k-1} - S_{k-1}^{k-1,0}) \\ C_k^{k-1} &= F_{k-1} C_{k-1}^{k-1} F_{k-1}^T + Q_{k-1} \end{aligned}$$

Multiple scattering
Energy loss

Use measurements to update state vector

$$\begin{aligned} K_k &= C_k^{k-1} H_k^T [V_k + H_k C_k^{k-1} H_k^T]^{-1} \\ S_k^k &= S_k^{k-1} + K_k (m_k - h(S_k^{k-1})) \\ C_k^k &= C_k^{k-1} - K_k H_k C_k^{k-1} \end{aligned}$$

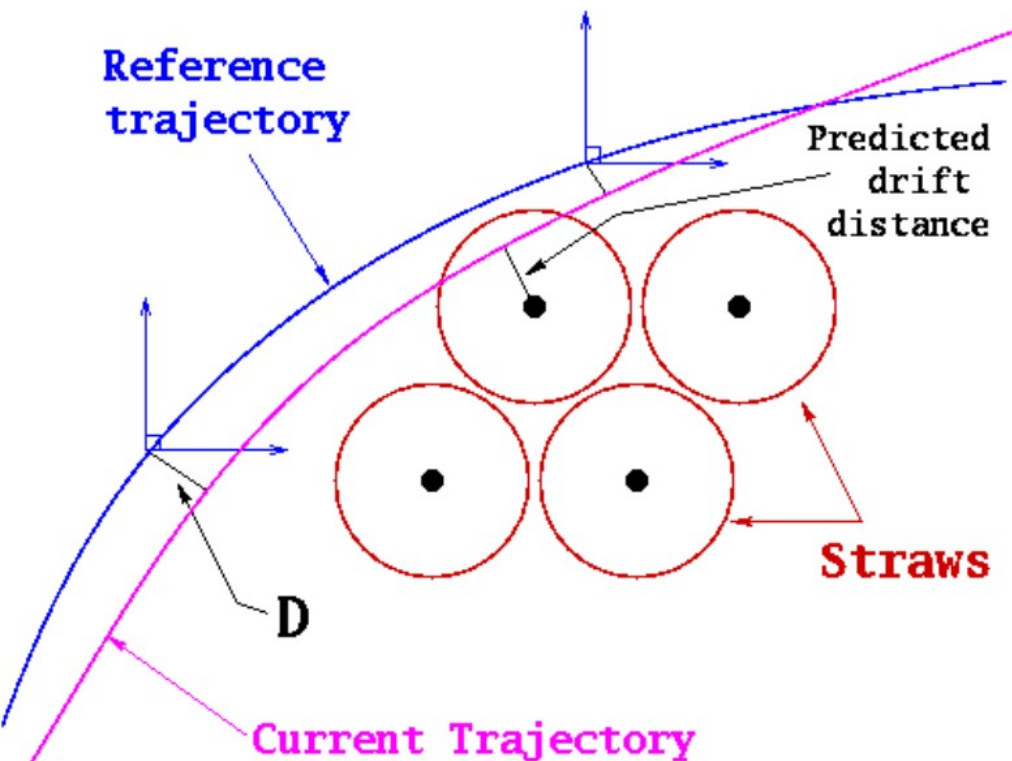
Use result (S) of iteration as new seed

Forward Tracking

- State vector $\{x, y, t_x = dx/dz, t_y = dy/dz, q/p\}$
 - “Fitted” state vector considered as small perturbation relative to a seed
 - Seed determined from list of track candidates using **helical model**
- First step: create **reference trajectory** from seed, swimming from vertex to most downstream FDC hit
 - Take into account multiple scattering and energy loss when stepping through the field – only do this for the reference trajectory...
- Measurements added one by one, starting with most downstream hit
- Iterate up to 20 times
 - Generally only 2 or 3 iterations needed for convergence
 - CDC hits included without changing state vector
- Initial guess for covariance matrix (off-diagonal elements=0)
 $\sigma(\delta p/p) = 10\%$, $\sigma_y = 3.3 \text{ mm}$, $\sigma_x = 3.3 \text{ mm}$, $\sigma_{t_x} = 100 \text{ mrad}$, $\sigma_{t_y} = 100 \text{ mrad}$

Central Tracking

- Start out by swimming a reference trajectory as for forward tracking



- Use central parameters:

$$\{q/p_T, \phi, \tan\lambda, D, z\}$$

- D = distance of closest approach to reference trajectory at a given time along the particle's path
- Origin of coordinate system for D moves from point to point along reference trajectory
- $\lambda = \pi/2 - \theta$ (“Dip angle”)

- Initial guess for covariance matrix (off-diagonal elements=0)

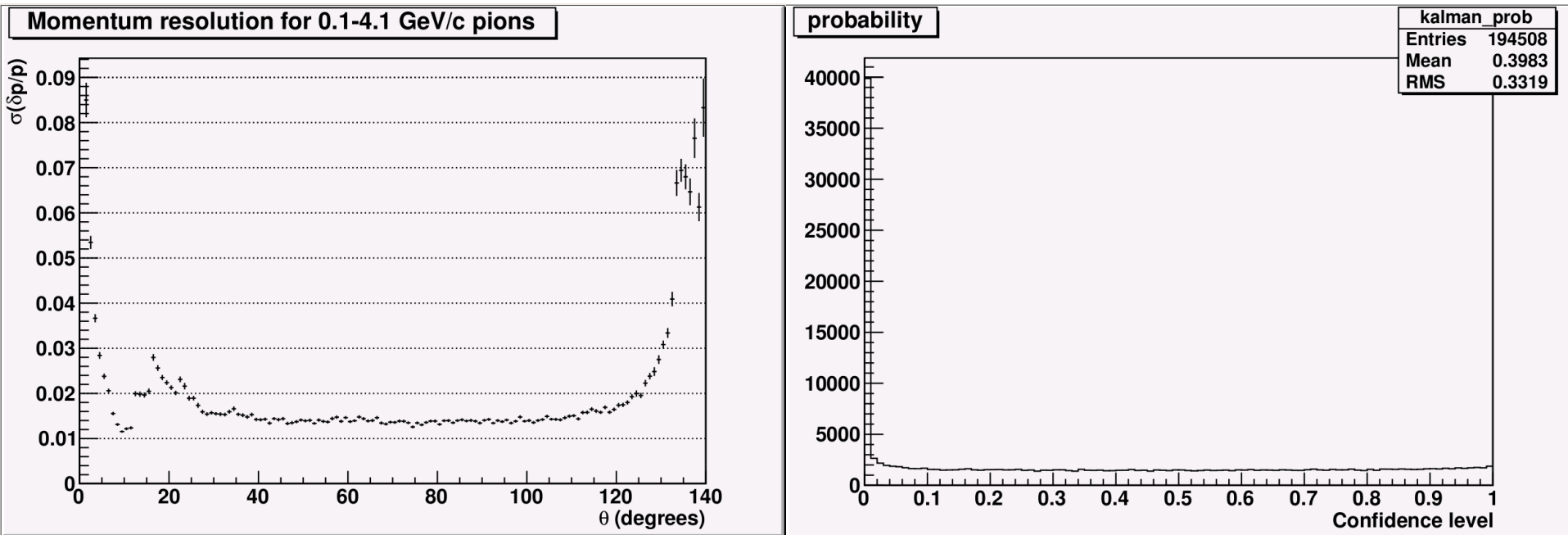
$$\sigma(\delta p_T/p_T) = 10\%, \sigma_D = 3.3 \text{ mm}, \sigma_z = 3.3 \text{ mm}, \sigma_\lambda = 100 \text{ mrad}, \sigma_\phi = 100 \text{ mrad}$$

Step 3: Time-based fitting

- Match wire-based track to Start Counter (or TOF or BCAL if no hit in SC) and propagate corrected SC (TOF, BCAL) time to “vertex”
→ T_0
 - Code for matching in *DParticleID* class
- Swim out from “vertex” and reselect hits belonging to track (using *DTrackHitSelector*)
- Refit using drift time information → *DTrackTimeBased*
 - Initial seed for parameters = result of wire-based fit

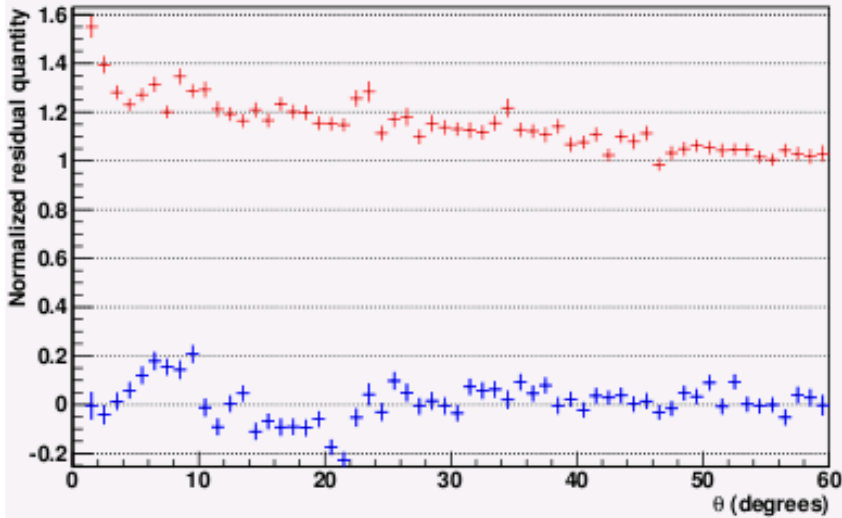
Fit quality

- Generated 0.1-4.1 GeV/c pions from center of target
 - Multiple-scattering and energy loss ON
 - Decays, secondary particles, hadronic interactions OFF

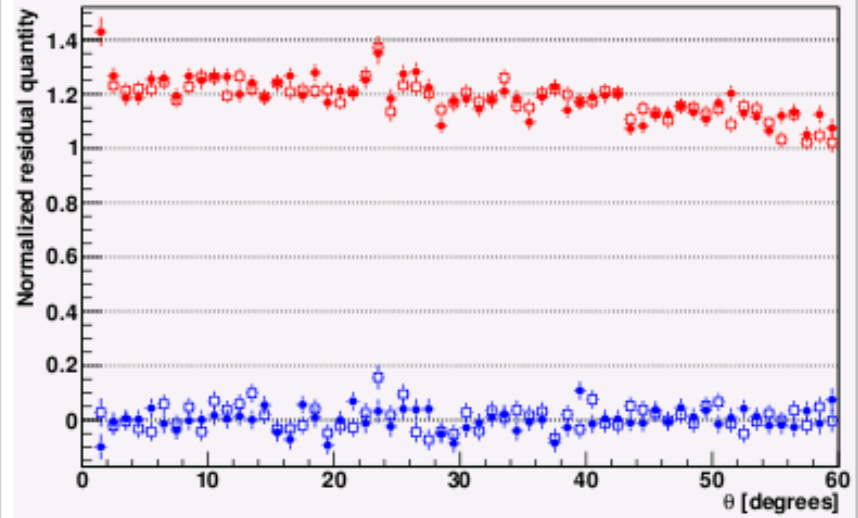


Normalized Residuals

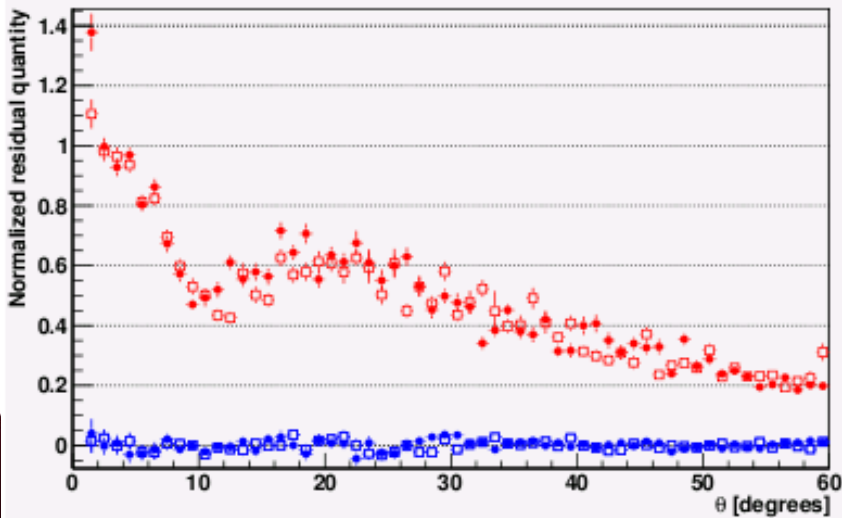
Mean residual and σ for q/p



Mean residual and σ for direction tangents



Mean residual and σ for positions

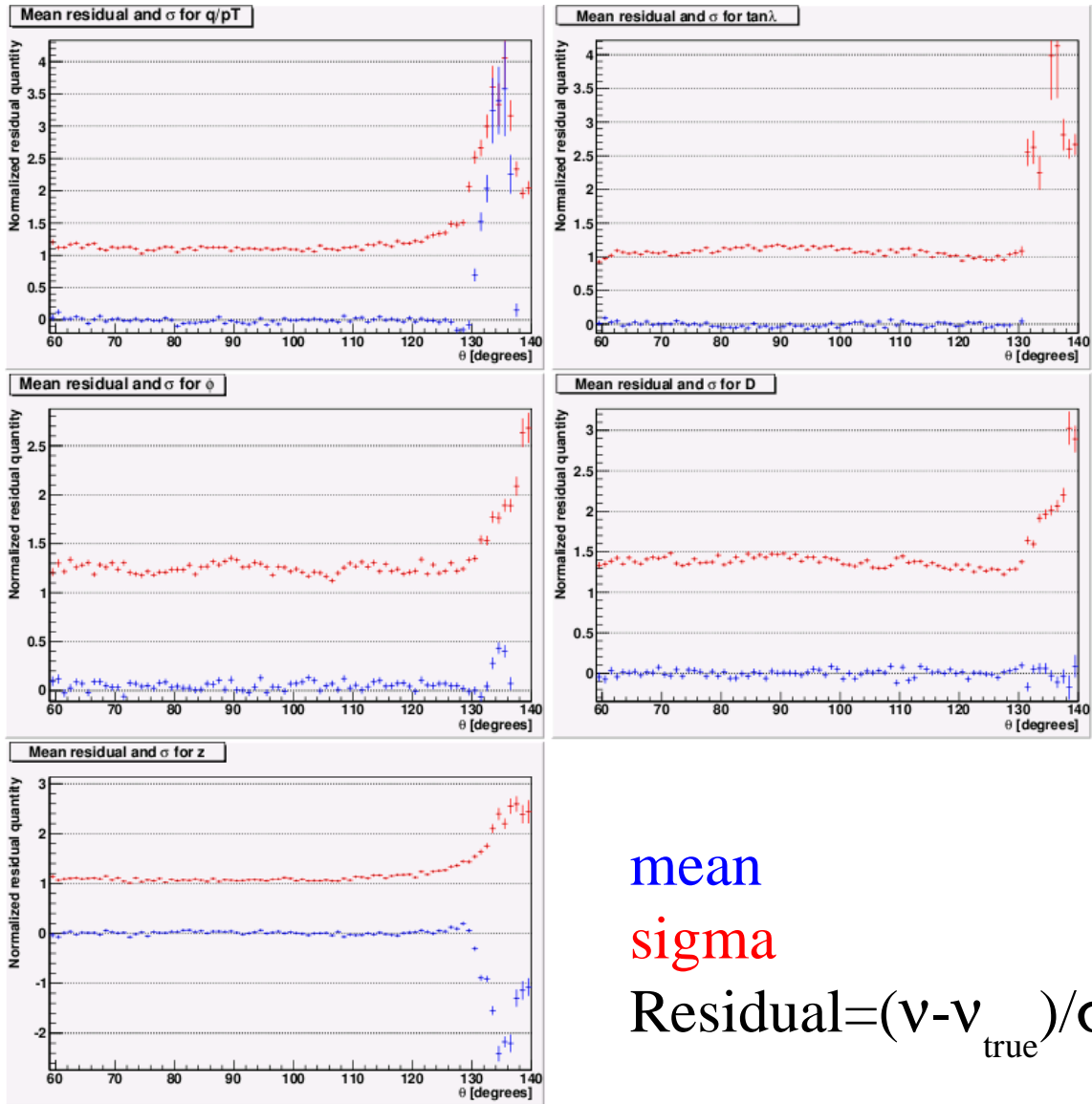


mean

sigma

$$\text{Residual} = (\mathbf{v} - \mathbf{v}_{\text{true}}) / \sigma_{\mathbf{v}}$$

Normalized Residuals



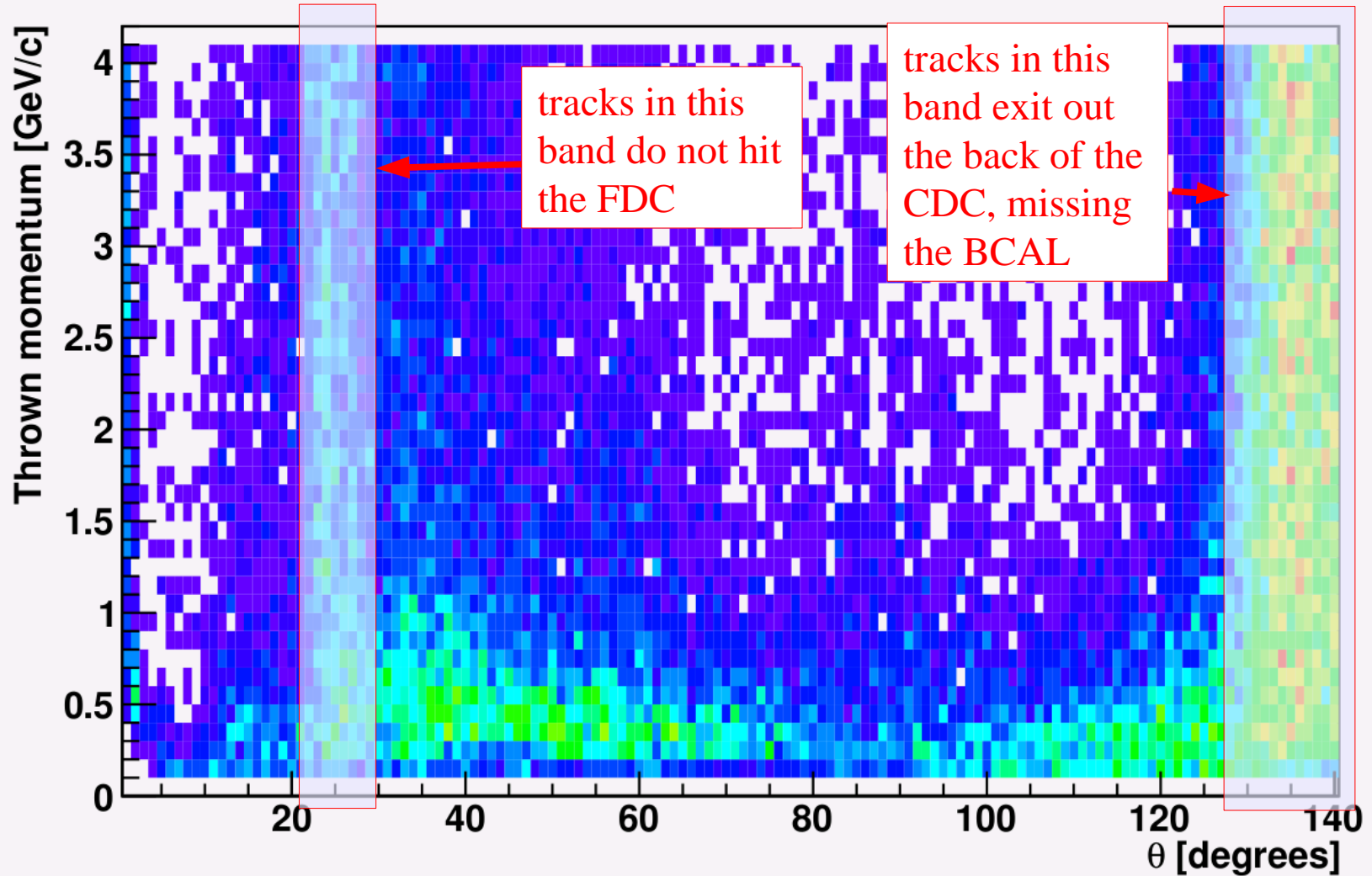
mean

sigma

$$\text{Residual} = (v - v_{\text{true}}) / \sigma_v$$

Phase space for bad-quality fits

Phase space for time-based fits with $CL < 0.001$



Issues

- Efficiency for single tracks is high but there are problem regions
- Poor quality fits leading to bands of low CL at certain angles and certain momenta need to be addressed
- Errors due to multiple scattering could be improved